PHYS-3301 Homework 2 Due 24 Sept 2014

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Angular Momentum** *Barton 2.9 plus*

   Angular momentum for a single particle is defined as $\vec{L} = \vec{x} \times \vec{p}$ in a given reference frame S.

   (a) Show that the value $\vec{L}'$ of the angular momentum measured in a reference frame S' is

      i. $\vec{L}' = \vec{L} - \vec{b} \times \vec{p}$ if S' is translated by $\vec{b}$ compared to S. (Meaning that angular momentum does depend on your choice of origin.)

      ii. $\vec{L}' = \vec{L} + \vec{v} \times (m\vec{x} - \vec{p}t)$ if S' is boosted by $\vec{v}$ compared to S.

   (b) Define the total angular momentum $\vec{\mathbf{L}} = \sum_i \vec{L}_i$ as the sum of the individual particle angular momenta in a system. Suppose $\vec{\mathbf{L}}$ is measured in the CM frame of the system with the origin chosen at the position of the center of mass. Show that the total angular momentum $\vec{\mathbf{L}}'$ measured in any frame S' that is translated and boosted with respect to S is equal to $\vec{\mathbf{L}}$ **plus the angular momentum of a particle with the total momentum and position of the CM in primed frame**.

2. **Swimming Up-River** *based on Hogg 1-4*

   You are on the west bank of a south-flowing river. The river flows at speed v past the ground, and you are able to swim at speed u in still water with $u > v$. Suppose you want to swim and land due east of your current location on the opposite bank of the river.

   (a) At what angle to the west-east axis must you aim yourself (and is it to the north or south)? Note: even though you are aiming differently, you end up swimming due east with respect to the ground.

   (b) What is your speed relative to the ground if you swim due east as described above?

   (c) Explain briefly how this calculation relates to (part of) the Michelson-Morley experiment.

3. **Global Positioning System**

   This problem will investigate how important relativistic effects are to one piece of technology that you may use in your daily life (like the Winnipeg Transit buses, which use GPS to identify the next stop). A GPS unit works by receiving time signals from multiple satellites and using that to calculate its position based on the distances to those satellites (which is given by c multiplied by the time the signal takes to reach the GPS from the satellite).

   (a) GPS satellites orbit at a height of approximately $r = 30,000$ km from the center of the earth. Find the ratio u/c of the orbital speed of the satellite to the speed of light, first in terms of Newton's constant G, the mass of the earth $M_\oplus$, and orbital radius r, and then as a pure number. *Reminder:* To one significant digit, $G = 7 \times 10^{-11}$ m^3/kg/s^2 and the mass of the earth is $M_\oplus = 6 \times 10^{24}$ kg.

   (b) GPS claims to be able to locate a receiver to within a radius of about 1 m. Considering that the distances used to locate a receiver are calculated by how long a light signal takes to travel between the satellite and the receiver, what error δt in the time signal sent by the satellite would translate into an error of $\delta r \approx$1 meter in position? You may work approximately, ignoring numerical factors of order one.

(c) At relative velocities $v \ll c$, show that

$$|t' - t| \approx \frac{v^2}{c^2} t \ , \tag{1}$$

where t is satellite time and t' is our time. If the GPS did not account for special relativity, this would be an error δt in the time signal as in part (b). How long would it take (elapsed time t) for δt to become large enough to give an error in location of more than a meter? *Note:* Once again, we are dropping numerical constants of order 1 and working with one significant digit.