PHYS-3301 Homework 11 Due 26 Nov 2014

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **The Cosmic Microwave Background**

   The universe is filled with left-over radiation from the Big Bang called the Cosmic Microwave Background, which has a typical frequency of $\bar{\omega} = 160$ GHz averaged over the sky. There is a smooth variation of frequency at different points on the sky. The maximum observed frequency is 200 MHz higher than average in one particular spot on the sky; the minimum frequency is 200 MHz lower than average at the diametrically opposite point. This effect is due to the motion of our galaxy relative to the overall rest frame of the microwave background.

   (a) What is the relative speed of the earth and the source of the Cosmic Microwave Background in km/s? Remember that $c \approx 3 \times 10^8$ m/s.

   (b) Suppose we measure the frequency at an angle θ on the sky from the maximum frequency point. Show that the frequency is

   $$\omega(\theta) = \bar{\omega} + (200 \text{ MHz}) \cos\theta \ . \tag{1}$$

2. **Fast Orbit**

   The star S-2 orbits the (presumed) black hole at the center of our galaxy with the highest known orbital speed of any astrophysical object. Consider a simplified model of its orbit as a perfect circle with a period of $T \approx 15$ yr and orbital speed of $u \approx 0.02c$. Also suppose that the earth is in the same plane as S-2's orbit at a distance much greater than the size of the orbit. S-2 emits a spectral line of frequency $\omega \approx 8 \times 10^{14}$ Hz; what is the difference of the observed frequency at earth to the emitted frequency as a function of emitted time t? Let $t = 0$ be the distance of closest approach between S-2 and earth. Give the formula first in terms of variables and then calculate numerical values. Work to 1 significant figure. You may safely assume that the earth's orbital speed around the sun and the sun's orbital speed around the center of the galaxy are negligible.

3. **Distribution Averages**

   Suppose that we have a number N of particles moving in 1D with distribution function

   $$f(x,p) = \frac{N\sqrt{ab}}{\pi} e^{-(ap^2+bx^2)} \ .$$

   (This is the thermal equilibrium distribution for particles moving in a harmonic oscillator potential.) Calculate the average values for the following quantities: (a) x (b) x^2 (c) xp (d) p^4.

4. **Phase Space** *from questions by Reif*

   Consider 2 particles moving in a box of length L in one dimension. The four phase space coordinates are x_1, x_2 (the positions of the two particles) and p_1, p_2 (the momenta of the two particles). The particles have total energy between E and E' ($E' > E$). Draw the (x_1, x_2) plane and the (p_1, p_2) plane and shade the area in the phase space allowed for the two particles (ideally, you would draw a four-dimensional phase space figure, but that's just too hard). Make sure to label your figures.