PHYS-3301 Homework 10 Due 19 Nov 2014

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Mandelstam Variables** *Based on Barton 11.8*

   Imagine a process where a particle of mass m_1 and 4-momentum p_1^μ collides with a particle of mass m_2 and 4-momentum p_2^μ. After the collision, there are particles of mass m_3 and momentum p_3^μ and mass m_4 and momentum p_4^μ. To describe this scattering process, particle physicists will often define the *Mandelstam variables* s, t, u as

   $$s = -(p_1 + p_2)^2 \ , \quad t = -(p_1 - p_3)^2 \ , \quad u = -(p_1 - p_4)^2 \ , \tag{1}$$

   where the square is the relativistic dot product of each 4-vector with itself.

   (a) Show that $s + t + u = (m_1^2 + m_2^2 + m_3^2 + m_4^2)c^2$. *Hint:* 4-momentum conservation allows you to write $p_3^\mu + p_4^\mu - p_2^\mu = p_1^\mu$.

   (b) Show that $s = (E^*/c)^2$, where E^* is the total energy in the CM frame.

2. **Reflected Sound** *based on Barton*

   Consider a police car situated to the left of a large tractor-trailer truck. The sound of the police siren reflects off the back of the truck and back to the police officer. In the police car's frame, the initial sound frequency is ω. In the two situations below, what is the frequency of the reflected wave that the police officer hears? *Hint:* In the rest frame of the reflecting surface, the reflected sound wave has the same frequency as the incident sound wave.

   (a) In a frame with no wind, the police car moves at speed $u < c_s$ toward the truck, which is stationary.

   (b) In a frame with no wind, the truck moves at speed $u < c_s$ toward the police car, which is stationary. If your answer is different than the previous part, explain.

3. **Spaceship Communications**

   Space station ES10 sits at galactic coordinates $x = 0$ lyr, $y \equiv y_0 = 3$ lyr. The station is able to receive radio signals in the range of $\omega_1 = 3$ MHz to $\omega_2 = 25/3$ MHz. Suppose that the experimental starship NX-02 travels along the x axis at constant speed $u = 4c/5$, passing $x = 0$ at $t = 0$ according to the spacestation. The starship broadcasts its positions at frequency $\omega_E = 5$ MHz. From what positions x can the space station receive the starship's signal?