PHYS-3301 Homework 1 Due 17 Sept 2014

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Rotations and Orthogonal Matrices**

   Two position vectors have the dot product $\vec{x} \cdot \vec{y} = x^i y^i$ (remember, superscripts tell you the component, and we use Einstein summation convention). The length of a vector $|\vec{x}|$ is given by the vector's dot product with itself as follows: $|\vec{x}|^2 = \vec{x} \cdot \vec{x}$.

   (a) Rotations leave the lengths of vectors invariant (unchanged). Prove that this means that dot products are invariant under rotations.

   (b) Use the invariance of all dot products under rotations to show that all rotation matrices R satisfy the identity
   $$R^i{}_j R^i{}_k = \delta_{jk} \ . \tag{1}$$
   Hint: To prove (1), consider frame S' rotated by R with respect to frame S. Then write the dot product $\vec{x}' \cdot \vec{y}'$ in terms of $\vec{x}, \vec{y}$ and set it equal to $\vec{x} \cdot \vec{y}$. You get an equation true for all $\vec{x}, \vec{y}$, which allows you to cancel the vector components from both sides of the equation.

   (c) Treat each column of R as a vector. Show that all the columns are perpendicular to each other and have length 1. This means that rotation matrices are *orthogonal matrices.*

2. **Choosing Frames Wisely**

   In both parts, clearly state what inertial reference frame you use to solve the problem.

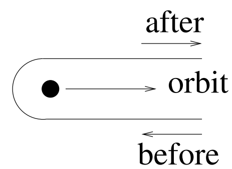
   (a) *from Barton 2.3* Train 1 of length L_1 moves right along a track with speed u_1, while train 2 of length L_2 moves left along a parallel track with speed u_2. Consider event A, the front of train 1 passes the front of train 2; event B, the front of train 1 passes the rear of train 2; and event C, the front of train 2 passes the rear of train 1. What are the lengths of time between events A and B and between events A and C? (*Hint:* first argue that these lengths of time are the same in all inertial frames.)

   (b) *Barton 2.10 rephrased* A cannonball is launched in an arc with velocity $\vec{u}$. At the top of its trajectory, a chemical charge in it explodes into two parts of masses m_1 and m_2 that separate in the horizontal direction only. The explosion releases energy E, which essentially all goes into the kinetic energy of the cannonball pieces. Show that they are separated by a distance $(u_y/g)\sqrt{2E(m_1+m_2)/m_1 m_2}$ when they land, where u_y is the initial vertical component of the velocity.

3. **Solar System Mechanics**

   (a) At some given moment in time, the moon moves at a speed of approximately 1 km/s in its orbit relative to the earth, and the earth-moon system moves at about 30 km/s in its orbit relative to the sun. The moon's mass is 7×10^{22} kg, the earth's mass is 6×10^{24} kg, and the sun's mass is 2×10^{30} kg. Calculate the total kinetic energy of the sun, earth, and moon as measured in their joint CM frame (in Joules). You may ignore all other matter in the solar system.

   (b) One strategy to help space probes like *Voyager* reach the outer solar system is to "sling-shot" around a planet. In the rest frame of the solar system as a whole, the planet moves in its orbit, while the probe starts out moving toward the planet opposite the orbital velocity. Then the probe whips around the planet and moves away in the same direction as the orbital velocity (see the figure).



Suppose the planet is Saturn, which has orbital speed about 10 km/s, and that our probe is initially moving at 3 km/s with respect to the sun before it encounters Saturn. What is the probe's speed after the slingshot? Treat the slingshot as an elastic collision and take the limit that Saturn's mass is infinite.