Gauge Fields & Strings Homework 3 Due 23 Oct 2013

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may instead email a PDF (prepared with LaTeX) to Dr. Frey.

1. **Nonabelian Gauge Transformation** *Srednicki 69.1*

2. **Quadratic Casimir** *Srednicki 69.2*
   The combination T^aT^a, which commutes with all generators, is known as the quadratic Casimir of the representation.

3. **The Little Group and String States**

   Consider a massless particle with momentum $p^+ \neq 0, p^- = p^I = 0$. The Lorentz transformations that leave these conditions invariant are called the *little group* of the particle. In this case, the little group consists of the boost along x^1 and the rotations among the transverse directions $x^{2,\dots D-1}$.

   (a) Show that the states of a photon (which is massless) transform as a vector under the little group rotations. See chapter 10 if necessary. Note that if one polarization exists, a little group rotation transforms it into another polarization.

   Now imagine a massless particle with states that transforms as a rank 2 tensor under the little group. This tensor can be split into an antisymmetric part $A^{IJ}(= -A^{JI})$, a symmetric traceless part $S^{IJ}(= S^{JI}, S^{II} = 0)$, and a trace part $\Phi^{IJ} = \Phi\delta^{IJ}/(D-2)$.

   (b) How many states are there of each type?

   (c) Use the tensor rotation transformation $T'^{IJ} = R^I_K R^J_L T^{KL}$ (for rotation matrix R) to show that the three types of states rotate only into themselves; that is, a transformed antisymmetric tensor stays antisymmetric, etc.

   (d) The antisymmetric tensor states must come from a field $B_{\mu\nu}(= -B_{\nu\mu})$ in the full Lorentz invariant theory. Show that the field strength

   $$H_{\mu\nu\lambda} \equiv \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu} \tag{1}$$

   is invariant under gauge transformations

   $$B_{\mu\nu} \to B_{\mu\nu} + \partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu \ . \tag{2}$$

   This *Kalb-Ramond* field is a generalization of the electromagnetic potential.

   Finally, consider a massive particle. In this case, the little group is the set of rotations among $x^{1,2,\cdots D-1}$ in the particle's rest frame ($p^0 \neq 0, p^1 = p^I = 0$).

   (e) How many polarization states are there in a vector of this little group?

   (f) Assuming that each transverse X^I gives the normal ordering constant $(1/2)\sum_{p=1}^{\infty} p = -1/24$ to the mass-squared operator for open strings,

   $$M^2 = \frac{1}{\alpha'}\left(\sum_{n=1}^{\infty} \alpha^I_{-n}\alpha^I_n - \frac{D-2}{24}\right) \ , \tag{3}$$

   argue that the states $\xi_I \alpha^I_{-1}|p^+, p^I\rangle$ must be massless so that the spacetime dimension is $D = 26$.