

PHYS-3103 Homework 1 Due 23 Oct 2013

This homework is due **in class** on the due date. You may instead email a PDF (prepared with LaTeX) to Dr. Frey in advance.

1. The Little Group and String States

Consider a massless particle with momentum $p^+ \neq 0, p^- = p^I = 0$. The Lorentz transformations that leave these conditions invariant are called the *little group* of the particle. In this case, the little group consists of the boost along x^1 and the rotations among the transverse directions $x^{2, \dots, D-1}$.

- (a) Show that the states of a photon (which is massless) transform as a vector under the little group rotations. See chapter 10 if necessary. Note that if one polarization exists, a little group rotation transforms it into another polarization.

Now imagine a massless particle with states that transforms as a rank 2 tensor under the little group. This tensor can be split into an antisymmetric part $A^{IJ} (= -A^{JI})$, a symmetric traceless part $S^{IJ} (= S^{JI}, S^{II} = 0)$, and a trace part $\Phi^{IJ} = \Phi \delta^{IJ} / (D - 2)$.

- (b) How many states are there of each type?
- (c) Use the tensor rotation transformation $T^{I'J'} = R_{K'}^I R_{L'}^J T^{KL}$ (for rotation matrix R) to show that the three types of states rotate only into themselves; that is, a transformed antisymmetric tensor stays antisymmetric, etc.
- (d) The antisymmetric tensor states must come from a field $B_{\mu\nu} (= -B_{\nu\mu})$ in the full Lorentz invariant theory. Show that the field strength

$$H_{\mu\nu\lambda} \equiv \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu} \quad (1)$$

is invariant under gauge transformations

$$B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu . \quad (2)$$

This *Kalb-Ramond* field is a generalization of the electromagnetic potential.

Finally, consider a massive particle. In this case, the little group is the set of rotations among $x^{1,2, \dots, D-1}$ in the particle's rest frame ($p^0 \neq 0, p^1 = p^I = 0$).

- (e) How many polarization states are there in a vector of this little group?
- (f) Assuming that each transverse X^I gives the normal ordering constant $(1/2) \sum_{p=1}^{\infty} p = -1/24$ to the mass-squared operator for open strings,

$$M^2 = \frac{1}{\alpha'} \left(\sum_{n=1}^{\infty} \alpha_{-n}^I \alpha_n^I - \frac{D-2}{24} \right) , \quad (3)$$

argue that the states $\xi_I \alpha_{-1}^I |p^+, p^I\rangle$ must be massless so that the spacetime dimension is $D = 26$.