PHYS-4601 Homework 7 Due 24 Oct 2013

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

In this entire assignment, the physical system studied is the 1D harmonic oscillator.

It may be useful to remember the Gaussian integral

$$\int_{-\infty}^{\infty} dx\, e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}\ . \tag{1}$$

1. **Half a Harmonic Oscillator** *Griffiths 2.42 rewritten*

   Consider the potential

   $$V(x) = \left\{ \begin{array}{cc} \infty & x \leq 0 \\ \frac{1}{2}m\omega^2 x^2 & x > 0 \end{array} \right. \ . \tag{2}$$

   What are the energy eigenvalues of this potential?

2. **Matrix Elements**

   Calculate the matrix elements $\langle n|x|m\rangle$ and $\langle n|p^2|m\rangle$ for $|n\rangle, |m\rangle$ stationary states of the harmonic oscillator. You *must* use Dirac and operator notation and *may not* carry out any integrals.

3. **Coherent States** *parts of Griffiths 3.35*

   In this problem, we will study *coherent states*, which are eigenfunctions of the lowering operator

   $$a|\alpha\rangle = \alpha|\alpha\rangle\ , \tag{3}$$

   where the eigenvalue α is generally complex.

   (a) Show that a coherent state can be written as

   $$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle\ . \tag{4}$$

   To do that, you will first want to show that $\alpha\langle n|\alpha\rangle = \sqrt{n+1}\langle n+1|\alpha\rangle$. That gives you a recursion relation that the series (4) satisfies. Then you can check that $|\alpha\rangle$ is normalized (remember that $|n\rangle$ are orthonormal).

   (b) Show that (4) is equivalent to

   $$|\alpha\rangle = e^{-|\alpha|^2/2} \exp\left[\alpha a^\dagger\right]|0\rangle\ . \tag{5}$$

   (c) If $|\alpha\rangle$ is the initial state of the system, show that the state at time t is still a coherent state with eigenvalue $\alpha(t) = \alpha e^{-i\omega t}$.