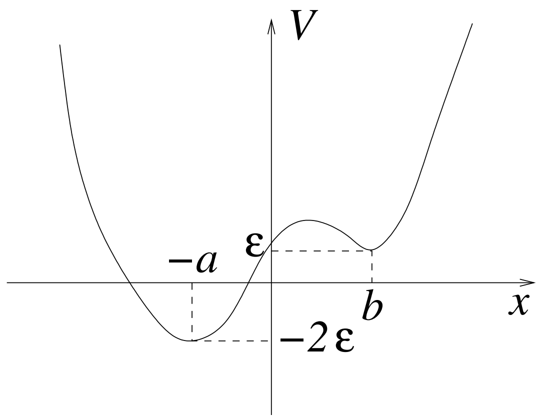
PHYS-4601 Homework 18 Due 13 Mar 2014

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Hand-Waving Variational Estimate**

   Consider the potential depicted in the figure below, which has local minima at $x = -a$ (where $V = -2\epsilon$) and $x = b$ ($V = \epsilon$). Near each of the two minima, the potential is well approximated by harmonic oscillator potentials with frequencies ω_a and ω_b respectively with $\omega_b > \omega_a$. Use a gaussian trial wavefunction of variable width and location to estimate the ground state energy using the variational principle. Also give a brief argument why your estimate overestimates the ground state energy based on the shape of the potential (not just the variational principle). *Do not do any calculations, just state your reasoning.*



2. **One More Variation**

   Consider a particle moving in 1D with potential $V(x) = (\hbar^2/2ma^2)(x/a)^4$, where a is constant with units of length. Write all the energies you find below in the form $E = \lambda(\hbar^2/2ma^2)$ where λ is a dimensionless number.

   (a) Use a Gaussian trial wavefunction (as discussed in the class notes) to find an upper bound on the ground state energy.

   (b) Choose your own trial wavefunction and find the corresponding upper bound on the ground state energy. You may use Maple (either to find a bound numerically or to help you find one analytically) if you attach your code and results. *Note:* I would use a smooth wavefunction probably, but you may use a piecewise defined one if you deal with the kinetic energy as in the reading from Griffiths.

   (c) Use the numerical method of assignment 6, problem 3 to find the ground state energy to within one percent error. How close are your upper bounds from parts (a,b)? Attach your Maple code.

3. **WKB as $\hbar$ Expansion** *Based on Griffiths 8.2*

   In this problem, you'll derive the WKB wavefunction in regions where $E > V(x)$. We will use the 1D Schrödinger equation in the form

$$\frac{d^2\psi}{dx^2} = -\frac{p(x)^2}{\hbar^2}\psi \ , \quad p(x) = \sqrt{2m(E - V(x))} \ . \tag{1}$$

(a) Begin by writing the wavefunction as $\psi(x) = \exp[if(x)/\hbar]$ for some complex function f (note that this is completely general). Use the Schrödinger equation (1) to find a second order differential equation for f.

(b) Now treat $\hbar$ as a small parameter, expanding $f(x) = f_0(x) + \hbar f_1(x) + \cdots = \sum_n \hbar^n f_n(x)$. Then write your differential equation from part (a) as a series in $\hbar$. Find the differential equations that come from the $\hbar^0$, $\hbar^1$, and $\hbar^2$ terms; these should vanish separately as $\hbar \to 0$.

(c) Find f_0 and f_1 in terms of $p(x)$, and use those to derive the WKB form of the wavefunction. *Hint:* Recall that $\ln z = \ln|z| + i\theta$, where the complex $z = |z|e^{i\theta}$.

(d) Describe how you would change this procedure for regions where $V(x) > E$.