PHYS-4601 Homework 13 Due 23 Jan 2014

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Quantum Mechanics In Your Face**

   Watch the video of Sidney Coleman's famous lecture at
   `http://media.physics.harvard.edu/video/?id=SidneyColeman_QMIYF`

2. **Finishing Bell's Inequality** *Griffiths 4.50*

   Consider 2 distinguishable spin 1/2 particles in the singlet ($s = 0$) total spin state. If $\vec{a}$ and $\vec{b}$ are two unit vectors, show that

   $$\left\langle \left(\vec{a} \cdot \vec{S}^{(1)} \right) \left(\vec{b} \cdot \vec{S}^{(2)} \right) \right\rangle = -\frac{\hbar^2}{4} \vec{a} \cdot \vec{b} \,. \tag{1}$$

   Hint: Think about a convenient choice of axes.

3. Three electrons are prepared in the so-called "GHZM" spin state $|\psi\rangle = (|+\rangle_1|+\rangle_2|+\rangle_3 - |-\rangle_1|-\rangle_2|-\rangle_3)/\sqrt{2}$ and distributed so that laboratories at locations A, B, and C each receive one electron.

   (a) Show that $|\psi\rangle$ is an eigenstate of the operator $S_x^{(1)} S_y^{(2)} S_y^{(3)}$ and find the eigenvalue.

   (b) If the electrons' total state is written as $|\phi\rangle \otimes |\psi\rangle$, where $|\phi\rangle$ is the spatial state of the electrons, what is $|\phi\rangle$? Each single-electron spatial state is $|A\rangle$, $|B\rangle$, or $|C\rangle$.

4. **White Dwarfs** *based on Griffiths 5.35*

   White dwarfs are old, dead stars that don't support nuclear fusion any more. Instead, electron degeneracy pressure keeps them from collapsing into black holes. In this problem, you'll work out the white dwarf radius by finding the minimum energy. *Note:* You will need equation [5.45] from Griffiths.

   (a) Imagine that the white dwarf is a sphere of uniform density. Caculate the gravitational potential energy V in terms of its radius R, total mass M, and Newton's constant G. *Hint*: To do this, work out the potential energy of a shell of thickness dr at radius r, which comes from the sphere of matter inside that shell. Then integrate from $r = 0 \rightarrow R$. You should find $V = -3GM^2/5R$.

   (b) Now treat the electrons as a free electron gas. Write their total energy (which is kinetic) in terms of M, R, the electron mass m, the nucleon mass m_N, the number of electrons per nucleon q, and $\hbar$. Then add this to the result of part (a) to get the total energy and find the radius R where the energy is minimized.

   (c) Assuming $q \approx 1/2$ (the average nucleus is a helium nucleus), find the radius (in km) of a white dwarf with the mass of the sun. You will need $G = 7 \times 10^{-11}$ m^3/kg/s^2, $\hbar = 1 \times 10^{-34}$ Js, $m = 9 \times 10^{-31}$ kg, $m_N = 2 \times 10^{-27}$ kg, and $M = 2 \times 10^{30}$ kg.

   (d) A neutron star is an extremely dense star where all the matter is neutrons, which is supported against collaps by the degeneracy pressure of the neutrons. In this case, $q = 1$. Find the ratio of the radius of a neutron star to the radius of a white dwarf of the same mass (assume $q \approx 1$ for the white dwarf), both in terms of physical constants and as a pure number to 1 significant digit.