PHYS-4601 Homework 1 Due 12 Sept 2013

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Some Practice with Dirac Notation**

   Consider the Hilbert space of L^2 functions on the interval $0 \leq x \leq 2\pi R$ with periodic boundary conditions. Using the fact that the complex exponentials $|e_n\rangle \simeq e^{inx/R}/\sqrt{2\pi R}$ form an orthonormal basis to carry out the following calculations *without doing any integrals.*

   (a) Calculate the inner product of $|f\rangle \simeq f(x) = \cos^3(x/R)$ and $|g\rangle \simeq g(x) = \sin(3x/R)$.

   (b) Find the inner product of $|f\rangle$ from part (a) with $|h\rangle \simeq \sin(3x/R + \theta)$.

   (c) $|f\rangle, |g\rangle, |h\rangle$ are not normalized. Find their norms.

2. **Polynomial Vector Spaces** *inspired by Griffiths A.2*

   Consider the set of all polynomials up to order N in x with complex coefficients. This problem introduces other formulations of vector spaces and inner products.

   (a) Give a brief argument (not necessarily a detailed proof) that these form a vector space. List a convenient basis for this vector space. What is the dimensionality?

   (b) The basis you listed for part (a) may or may not be orthonormal with respect to any inner product. Now consider $-1 \leq x \leq 1$. Show explicitly that the first three Legendre polynomials given in Griffiths table 4.1 (P_0, P_1, P_2) are orthogonal with respect to the usual L^2 inner product. In fact, the Legendre polynomials form an orthogonal basis.

   (c) Next, show that the first three Laguerre polynomials for $0 \leq x < \infty$ as defined in Griffiths table 4.5 (L_0, L_1, L_2) are orthogonal with respect to the inner product

   $$\langle f|g\rangle = \int_0^\infty dx\, e^{-x} f^*(x) g(x) \ . \tag{1}$$

3. **Something We'll Call "Momentum"**

   In class, we defined states $|x\rangle$ corresponding to a particle at a well-defined position. Now define states $|p\rangle$ with wavefunctions

   $$\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar} \ . \tag{2}$$

   For simplicity, stick to one dimension.

   (a) Show that $\langle p'|p\rangle = \delta(p - p')$. *Hint:* You may find the formula

   $$\delta(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{ikz} \tag{3}$$

   helpful.

   (b) Show that the wavefunction $\psi(x) = \langle x|\psi\rangle$ and "momentum-space wavefunction" $\tilde{\psi}(p) = \langle p|\psi\rangle$ for any state ψ are Fourier transforms, as defined in Griffiths equation [2.102] (up to factors of $\hbar$). To work this out precisely, it will be helpful for you to rescale x and p to remove explicit powers of $\hbar$.

   (c) Consider a vector we call $|p \cdot \psi\rangle$ with momentum-space wavefunction $p\tilde{\psi}(p)$ made from a general state $|\psi\rangle$. Show that

   $$\langle x|p \cdot \psi\rangle = -i\hbar \frac{d\psi}{dx}(x) \ . \tag{4}$$