- Collisions, Decays, etc: Putting things Into practice
  - The idea will be to
    1) Use relativistic invariants / dot products to simplify calculations
    2) Use information from the most convenient (often CM) frame
  - Often, a problem / calculation will call for the initial rest frame of decaying particle or CM frame of 2+ particle collision
  - Our first two examples will show an "obvious" way (according to book) and the relativistic invt way
  - Example 1: 2 Body Decay at Rest

    A particle of mass M decays into particles of mass m_1 and m_2. ($\Lambda \to p + \pi^-$)
    Find energy E_1^* and momentum p_1^* of particle w/mass m_1 in initial rest frame
    - "Separate" Energy and Momentum conservation
      + First, note that $\vec{p}_1^* = -\vec{p}_2^*$. Call the common magnitude p^*
      + You could write that the total energy $Mc^2 = E_1^* + E_2^*$

$$Mc^2 = \sqrt{(p^*c)^2 + m_1^2c^4} + \sqrt{(p^*c)^2 + m_2^2c^4}$$

      Then do a bunch of algebra to find p^*, then E_1
      + A little easier is $E_2^* = \sqrt{(p^*c)^2 + (m_2c^2)^2} = \sqrt{(E_1^*)^2 - (m_1c^2)^2 + (m_2c^2)^2}$

      And $Mc^2 = E_1^* + E_2^* \Rightarrow (Mc^2 - E_1^*)^2 = E_1^{*2} - (m_1c^2)^2 + (m_2c^2)^2$

      Solve
$$E_1^* = (M^2 + m_1^2 - m_2^2)c^2/2M \text{ then find } p^*.$$
- 4-vector momentum conservation
  + We know that the initial 4-momentum P^μ is conserved
$$P^\mu = p_1^\mu + p_2^\mu.$$
  + If we subtract and square, $p_2^2 = (P - p_1)^2$ or
$$m_2^2c^2 = M^2c^2 + m_1^2c^2 + 2P\cdot p_1$$
  + We can evaluate the scalar product in CM frame
$$P\cdot p_1 = -(Mc)(E_1^*/c).$$ Gives right result.

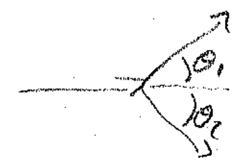
- Example 2: Moving 2 body decay

A π^0 particle of mass m moving at speed u relative to the lab decays into 2 photons that move off at angles θ_1, θ_2 to initial flight path. How does θ_1 depend on E_1?

- Separate Energy + Momentum Conservation
  + For a photon, the magnitude of momentum is E/c
  + Energy conservation says $\gamma mc^2 = E_1 + E_2$
  + Momentum conservation says:

Along velocity $\gamma mu = (E_1/c)\cos\theta_1 + (E_2/c)\cos\theta_2$

Perpendicular $(E_1/c)\sin\theta_1 = (E_2/c)\sin\theta_2$.



  + First eliminate θ_2, then E_2.

$$(\gamma mc^2 - E_1)^2 = (\gamma muc)^2 + E_1^2 - 2\gamma mucE_1\cos\theta_1$$

After massaging

$$E_1 = \frac{mc^2}{2\gamma\left(1 - \frac{u}{c}\cos\theta_1\right)}$$

- 4-momentum Conservation
  + Again, use $p_2^\mu = P^\mu - p_1^\mu$
  + Squaring, $0 = -m^2c^2 - 2P\cdot p_1$ or $m^2c^2 = 2\gamma m(E_1 - uE_1/c\cos\theta_1)$

Easy and direct

- Example 3: Production Thresholds

2 particles collide. What's the minimum energy needed to create some other set of particles? A very important question in expt design.

- A preliminary:
  + Total final state 4-momentum is P_f^μ. Define $P_f^2 = -M^2c^2$.
  + But in CM frame $\vec{P}_f^* = 0$, $P_f^{0*} = \sum_n E_n/c > \sum_n m_n c$

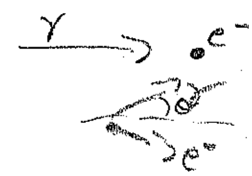
That means $(P_f^*)^2 \leq -(\sum m_n c)^2 \Rightarrow M \geq \sum_n m_n \leftarrow$ valid in any frame

- CM frame: What's the minimum CM frame energy needed?
  + $\vec{P}_A^* + \vec{P}_B^* = 0$. Therefore, $(P_A + P_B)^2 = -(E_A^* + E_B^*)^2/c^2$
  + 4 momentum conservation: $(E_A^* + E_B^*)^2 = M^2c^4 \geq (\sum m_n)^2 c^4$

- Lab frame: Suppose m_1 is moving but m_2 is stationary.
  - Then $-M^2c^2 = (p_A+p_B)^2 = -m_A^2c^2 - m_B^2c^2 + 2p_A\cdot p_B$
    Since $p_B^\mu = (m_Bc, \vec{0})$, $p_A \cdot p_B = -(m_Bc)(E_A/c) = -m_BE_A$
  - Therefore,
    $$E_A = \frac{1}{2m_B}(M^2 - m_A^2 - m_B^2)c^2 \geq \frac{1}{2m_B}\left[(\Sigma m_n)^2 - m_A^2 - m_B^2\right]c^2$$
  - This shows why Tevatron, LHC collide 2 moving beams (cm frame)
    At large M^2, $E_A^* \propto \Sigma m_n$ but stationary target $E_A \propto (\Sigma m_n)^2$.

- Example 4: The Compton Effect

Light strikes a stationary electron. Classically, wavelength is unchanged. Relativistically, we must include electron recoil.



- Electron momenta are p_μ, p'_μ. Photon momenta are q_μ, q'_μ.
  - Conservation $p'^\mu = p^\mu + q^\mu - q'^\mu \Rightarrow -m^2c^2 = (p+q-q')^2$
    $= -m^2c^2 + 2p\cdot(q-q') - 2q\cdot q'$
  - In this frame, $p^\mu = (mc, \vec{0})$, $q^\mu = (E/c, 0, 0, E/c)$,
    $q'^\mu = (E'/c, 0, E'/c \sin\theta, E'/c \cos\theta)$
  - Finally,
    $$m(E-E') = (1/c^2)EE'(1-\cos\theta)$$
- In quantum mechanics, photon energy is $E = hc/\lambda$.
  Therefore, the light changes wavelength h = Planck's constant
  $$\lambda' - \lambda = (h/mc)(1-\cos\theta)$$
  - h/mc is called the electron's Compton wavelength