PHYS-3301 Homework 9 Due 13 Nov 2013

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Electron Absorbing a Photon**

   Imagine that an electron (mass m) is hit by a photon of energy E and absorbs it completely. The final state of the system is just an electron, now moving off in some direction. You will prove in two different ways that this is *impossible* according to conservation of 4-momentum.

   (a) *First method:* Consider the reference frame where the final electron is at rest. In this frame, what is the energy of the final electron? Is the energy of the initial electron greater or less than this amount (assuming the photon has nonzero 4-momentum)? You should see that energy cannot be conserved in this frame, which means 4-momentum is not conserved in this or any other frame.

   (b) *Second method:* Conservation of 4-momentum says $P_i^\mu = P_f^\mu$, where P_i is the total initial momentum and P_f is the total final momentum. Show by taking the Lorentz-invariant square of this equation that 4-momentum can be conserved only if the initial photon energy is $E = 0$ in the rest frame of the initial electron. This means that there is no photon; for massless particles, $E = 0$ in one frame means that $E = 0$ in all frames (see previous assignments).

2. **Mandelstam Variables** *Based on Barton 11.8*

   Imagine a process where a particle of mass m_1 and 4-momentum p_1^μ collides with a particle of mass m_2 and 4-momentum p_2^μ. After the collision, there are particles of mass m_3 and momentum p_3^μ and mass m_4 and momentum p_4^μ. To describe this scattering process, particle physicists will often define the *Mandelstam variables* s, t, u as

   $$s = -(p_1 + p_2)^2 \ , \quad t = -(p_1 - p_3)^2 \ , \quad u = -(p_1 - p_4)^2 \ , \tag{1}$$

   where the square is the relativistic dot product of each 4-vector with itself.

   (a) Show that $s + t + u = (m_1^2 + m_2^2 + m_3^2 + m_4^2)c^2$. *Hint:* 4-momentum conservation allows you to write $p_3^\mu + p_4^\mu - p_2^\mu = p_1^\mu$.

   (b) Show that $s = (E^*/c)^2$, where E^* is the total energy in the CM frame.

3. **Inverse Compton Scattering**

   A photon of energy ϵ strikes an electron of energy E and mass m head on (that is, the spatial parts of their momenta are opposite each other). Call the initial 4-momentum of the photon q^μ and of the charged particle p^μ.

   (a) Suppose that the final photon moves back along the original photon's path. Find the energy ϵ' of the final photon.

   (b) Suppose E is very large, so you can ignore the electron mass. Use your answer from part (a) to show that $\epsilon' = E$ in this limit. (This shows that "inverse Compton scattering" can increase photon energies, since the photon steals essentially all the electron's energy.)

4. **"Non-relativistic" Annihilation**

Dark matter is a hypothetical particle that would interact with normal matter mainly through gravity. However, the recent discovery of an unusual gamma ray (photon) signal from the galactic center suggests that two dark matter particles can annihilate into two photons.

(a) If the dark matter mass is M and each dark matter particle has CM frame speed u, what is the energy of each outgoing photon in the limit that $u \to 0$?

(b) Now suppose the two dark matter particles can annihilate into a photon and a Z boson (one of the particles that carries the weak nuclear force). If the Z mass is m_Z, what is the energy of the photon in the CM frame in terms of M and m_z? Again take the limit that the dark matter speed $u \to 0$. *Hint:* despite the fact that the dark matter is "non-relativistic," use 4-vector momentum conservation and then use the fact that the dark matter particles are at rest in the CM frame.