PHYS-3301 Homework 5 Due 16 Oct 2013

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **A Moving Object** *Barton 6.6 plus*

   A relativistic car moves along the x axis, passing $x = 0$ at $t = 0$. Its velocity is $u(t) = dx/dt = c/\cosh(\omega t)$. **Important:** In the following, you will find some integrals involving hyperbolic trig functions. You may use computer programs such as Maple or Mathematica to do them ***only*** if you *cite the program you use* and *check the result of the indefinite integral by differentiating it and seeing that you get the integrand.*

   (a) Sketch the worldline of the car on a spacetime diagram. Can a real car follow this path exactly?

   (b) Find the relation between the time t and the car's proper time τ. Choose integration constants so that $\tau = 0$ when $t = 0$. Be careful taking signs if you take a square root.

   (c) Find u as a function of τ and $dx/d\tau$. *Hint:* Remember that $u = dx/dt$, not $dx/d\tau$.

   (d) Since the car moves, it doesn't experience as much proper time as coordinate time t. Find the total time lag Δ over all time, which is defined as

   $$\Delta = \lim_{t\to\infty} [t - \tau(t)] - \lim_{t\to-\infty} [t - \tau(t)] \ .$$

2. **Velocity Addition**

   In the lecture notes, we argued that a boost by v_1 along x followed by a boost of v_2 also along x is the same as a boost along x by

   $$v_3 = \frac{v_1 + v_2}{1 + v_1 v_2/c^2} \ , \tag{1}$$

   but we didn't finish the proof.

   (a) We needed to show that $\gamma(v_3) = \gamma(v_1)\gamma(v_2)(1 + v_1 v_2/c^2)$. To do this, prove the following:

   $$1 - \frac{v_3^2}{c^2} = \frac{(1 - v_1^2/c^2)(1 - v_2^2/c^2)}{(1 + v_1 v_2/c^2)^2} \ , \tag{2}$$

   using v_3 as given in (1). This is $1/\gamma(v_3)^2$, so it proves what we want.

   (b) Give a quick argument relating this velocity addition to the transformation of particle velocities to prove (1) indirectly.

3. **Ultrarelativistic Velocity Addition** *from Hogg 4.8*

   A neutral pion particle (π^0) of mass M is produced at rest with respect to the lab frame. In one possible but rare decay, it produces an electron and positron (anti-electron) which move off in opposite directions, each with mass m and relativistic γ factor of $\gamma = M/2m \approx 100$.

   (a) Since γ is so large, the speed of the electron or positron relative to the lab can be written as $u/c = 1 - \epsilon$. Find ϵ to the lowest order in the small number m/M (that is, if ϵ is written as a power series in m/M, find the power series out to the lowest power with a nonzero coefficient) and then to 1 significant digit.

   (b) Relative to the electron, what is the positron's speed? Again, write the relative speed as $u/c = 1 - \tilde{\epsilon}$ and find $\tilde{\epsilon}$ to lowest order in m/M and to 1 significant digit.