PHYS-3301 Homework 3 Due 2 Oct 2013

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Invariance of Light Speed**

   In this question, we'll show that the speed of light is invariant, no matter the direction of the velocity. (Of course, our derivation of the Lorentz transformations showed this for light moving along the relative motion of two frames.) Start in the S frame, where a light beam leaves the origin at time $t = 0$ and reaches point $\vec{x} = (x_0, y_0, 0)$ at time $t = \sqrt{x_0^2 + y_0^2}/c$, hitting a detector there. Now consider a frame S' moving at speed v relative to S along the x axis.

   (a) At what coordinates t', x', y', z' does the light hit the detector? Write your answer in terms of c, x_0, y_0, and v.

   (b) Find the components of the light's velocity in the S' frame by dividing x'/t', etc. What is the speed of light in S'?

   (c) Why don't we need to think about a component of motion along the z axis to get a general proof of the invariance of the speed of light?

2. **Adventures of SpaceKid** *somewhat inspired by Barton 4.2 & 4.3*

   A young explorer, age 8, leaves earth on an extremely fast rocket.

   (a) He or she travels first to alpha Centauri and arrives 5 years later Earth time and 4 lightyears away by Earth distance. How old is our explorer on arrival at alpha Centauri?

   (b) Immediately after, he or she leaves for epsilon Eridani, arriving 12 years later Earth time at age 16. How far apart are alpha Centauri and epsilon Eridani in the Earth's reference frame?

3. **Global Positioning System**

   This problem will investigate how important relativistic effects are to one piece of technology that you may use in your daily life (like the Winnipeg Transit buses, which use GPS to identify the next stop). A GPS unit works by receiving time signals from multiple satellites and using that to calculate its position based on the distances to those satellites (which is given by c multiplied by the time the signal takes to reach the GPS from the satellite).

   (a) GPS satellites orbit at a height of approximately $r = 30,000$ km from the center of the earth. Find the ratio u/c of the orbital speed of the satellite to the speed of light, first in terms of Newton's constant G, the mass of the earth $M_\oplus$, and orbital radius r, and then as a pure number. *Reminder:* To one significant digit, $G = 7 \times 10^{-11}$ m^3/kg/s^2 and the mass of the earth is $M_\oplus = 6 \times 10^{24}$ kg.

   (b) GPS claims to be able to locate a receiver to within a radius of about 1 m. Considering that the distances used to locate a receiver are calculated by how long a light signal takes to travel between the satellite and the receiver, what error δt in the time signal sent by the satellite would translate into an error of $\delta r \approx$1 meter in position? You may work approximately, ignoring numerical factors of order one.

   (c) At relative velocities $v \ll c$, show that

$$|t' - t| \approx \frac{v^2}{c^2} t \ , \tag{1}$$

where t is satellite time and t' is our time. If the GPS did not account for special relativity, this would be an error δt in the time signal as in part (b). How long would it take (elapsed time t) for δt to become large enough to give an error in location of more than a meter? *Note:* Once again, we are dropping numerical constants of order 1 and working with one significant digit.