PHYS-3301 Homework 1 Due 18 Sept 2013

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Rotations and Orthogonal Matrices**

   Two position vectors have the dot product $\vec{x} \cdot \vec{y} = x^i y^i$ (remember, superscripts tell you the component, and we use Einstein summation convention). The length of a vector $|\vec{x}|$ is given by the vector's dot product with itself as follows: $|\vec{x}|^2 = \vec{x} \cdot \vec{x}$.

   (a) Rotations leave the lengths of vectors invariant (unchanged). Prove that this means that dot products are invariant under rotations.

   (b) Use the invariance of all dot products under rotations to show that all rotation matrices R satisfy the identity

   $$R^i{}_j R^i{}_k = \delta_{jk} \ . \tag{1}$$

   Hint: To prove (1), consider frame S' rotated by R with respect to frame S. Then write the dot product $\vec{x}' \cdot \vec{y}'$ in terms of $\vec{x}, \vec{y}$ and set it equal to $\vec{x} \cdot \vec{y}$. You get an equation true for all $\vec{x}, \vec{y}$, which allows you to cancel the vector components from both sides of the equation.

   (c) Treat each column of R as a vector. Show that all the columns are perpendicular to each other and have length 1. This means that rotation matrices are *orthogonal matrices*.

2. **CM Frame**

   Consider n particles (labeled $i = 1, \ldots n$) in motion and define the total mass $M = \sum_i m_i$ and center of mass velocity $\vec{U} = (1/M) \sum_i m_i \vec{u}_i$.

   (a) First, show that the difference between two velocities is invariant under boost transformations. Then write the CM frame velocity $\vec{u}_i'$ of each particle as the difference of two velocities in our given lab frame.

   (b) Show that the total kinetic energy can be written as $K_{tot} = \vec{P}^2/2M + K_{int}$, where $\vec{P} = M\vec{U}$ is the total momentum and K_{int} is the kinetic energy measured in the CM frame. Write K_{int} solely in terms of quantities that are invariant under Galilean transformations, which proves that it is invariant.

3. **Choosing Frames Wisely**

   In both parts, clearly state what inertial reference frame you use to solve the problem.

   (a) *from Barton 2.3* Train 1 of length L_1 moves right along a track with speed u_1, while train 2 of length L_2 moves left along a parallel track with speed u_2. Consider event A, the front of train 1 passes the front of train 2; event B, the front of train 1 passes the rear of train 2; and event C, the front of train 2 passes the rear of train 1. What are the lengths of time between events A and B and between events A and C? (*Hint:* first argue that these lengths of time are the same in all inertial frames.)

   (b) *Barton 2.10 rephrased* A cannonball is launched in an arc with velocity $\vec{u}$. At the top of its trajectory, a chemical charge in it explodes into two parts of masses m_1 and m_2 that separate in the horizontal direction only. The explosion releases energy E, which essentially all goes into the kinetic energy of the cannonball pieces. Show that they are separated by a distance $(u_y/g)\sqrt{2E(m_1 + m_2)/m_1 m_2}$ when they land, where u_y is the initial vertical component of the velocity.