

PHYS-4601 Homework 7 Due 25 Oct 2012

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

In this entire assignment, the physical system studied is the 1D harmonic oscillator.

It will be useful to remember the Gaussian integral

$$\int_{-\infty}^{\infty} dx e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}. \quad (1)$$

1. Wavefunctions of the Harmonic Oscillator *Mostly Griffiths 2.10*

The text uses the condition that the ground state is annihilated by the lowering operator ($a|0\rangle = 0$) to find the ground state wavefunction $\langle x|0\rangle$. It also finds $\langle x|1\rangle$, the first excited wavefunction.

- Apply the raising operator a^\dagger to find the wavefunction $\langle x|2\rangle$ of the second excited state.
- First show that the ground state and second excited state are orthogonal by using the ladder operators. Then carry out the same calculation by direct integration of the wavefunctions.
- Griffiths 2.31(a)* Consider a state with wavefunction

$$\langle x|\psi\rangle = A \left(1 - 2\sqrt{\frac{m\omega}{\hbar}}x \right)^2 e^{-m\omega x^2/2\hbar} \quad (2)$$

for A constant. What is the expectation value of the energy. *Hint:* You will need the wavefunctions of the first few states of the harmonic oscillator. Remember that there is a unique way to write any state as a superposition of energy eigenstates.

2. Coherent States *Griffiths 3.35, roughly*

In this problem, we will study *coherent states*, which are eigenfunctions of the lowering operator

$$a|\alpha\rangle = \alpha|\alpha\rangle, \quad (3)$$

where the eigenvalue α is generally complex.

- Is there any energy eigenstate that is a coherent state? If so, list which energy eigenstate(s) are coherent and give the eigenvalue(s).
- Find the expectation values of x and p in the coherent state $|\alpha\rangle$ (use the ladder operators).
- Then find the uncertainties of x and p in $|\alpha\rangle$ and show that any coherent state is a state of minimal uncertainty.
- Show that a coherent state can be written as

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \quad (4)$$

To do that, you will first want to show that $\alpha\langle n|\alpha\rangle = \sqrt{n+1}\langle n+1|\alpha\rangle$. That gives you a recursion relation that the series (4) satisfies. Then you can check that $|\alpha\rangle$ is normalized (remember that $|n\rangle$ are orthonormal).

(e) Show that (4) is equivalent to

$$|\alpha\rangle = e^{-|\alpha|^2/2} \exp[\alpha a^\dagger] |0\rangle . \quad (5)$$

(f) If $|\alpha\rangle$ is the initial state of the system, show that the state at time t is still a coherent state with eigenvalue $\alpha(t) = \alpha e^{-i\omega t}$.