PHYS-4601 Homework 21 Due 4 Apr 2013

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Decay Rate of Excited Hydrogen**

   In this problem, you will find the decay rate for hydrogen going from the $n = 2$, $\ell = 1$ states to the $n = 1$, $\ell = 0$ ground state. We see in the lecture notes that the decay rate from a state $|\psi_i\rangle = |2, 1, m\rangle$ to $|\psi_f\rangle = |1, 0, 0\rangle$ by single photon emission is

   $$\Gamma = \frac{4\alpha}{3} \frac{\Delta E^3}{\hbar^3 c^2} \left|\langle \psi_f | \vec{x} | \psi_i \rangle\right|^2 \ , \tag{1}$$

   where α is the fine structure constant and $\Delta E = E_{n=2} - E_{n=1}$. This is summed over both polarizations and all outgoing photon momenta.

   (a) Show that $|\langle 1, 0, 0 | \vec{x} | 2, 1, 0 \rangle|^2 = |\langle 1, 0, 0 | \vec{x} | 2, 1, \pm 1 \rangle|^2$, so the decay rate is the same for any of the excited states. *Hint:* From selection rules discussed in the notes, the nonzero matrix elements under consideration are $\langle 2, 1, 0 | z | 1, 0, 0 \rangle$, $\langle 2, 1, \pm 1 | x | 1, 0, 0 \rangle$, and $\langle 2, 1, \pm 1 | y | 1, 0, 0 \rangle$. Find a relationship between these matrix elements by writing z in terms of $[L_\pm, x]$ and $[L_\pm, y]$, where $L_\pm$ are angular momentum raising and lowering operators.

   (b) Evaluate $|\langle 1, 0, 0 | \vec{x} | 2, 1, 0 \rangle|^2$ (you should only need to do a single integral) and find the decay rate Γ in terms of $\alpha, m, \hbar, c$. What is the lifetime $\tau \equiv 1/\Gamma$ in seconds?