PHYS-4601 Homework 18 Due 14 Mar 2013

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Variational Principle and Perturbation Theory** *from Griffiths 7.5*

   Consider a Hamiltonian $H = H_0 + H_1$, where H_0 is exactly solvable and H_1 is small in some sense. Prove that first-order perturbation theory always overestimates the true ground state energy. That is, show that the ground state energy calculated in first-order perturbation theory is greater than (or equal to) the true ground state energy. *Hint*: Think about the variational principle.

2. **One More Variation**

   Consider a particle moving in 1D with potential $V(x) = (\hbar^2/2ma^2)(x/a)^4$, where a is constant with units of length. Write all the energies you find below in the form $E = \lambda(\hbar^2/2ma^2)$ where λ is a dimensionless number.

   (a) Use a Gaussian trial wavefunction (as discussed in the class notes) to find an upper bound on the ground state energy.

   (b) Choose your own trial wavefunction and find the corresponding upper bound on the ground state energy. You may use Maple (either to find a bound numerically or to help you find one analytically) if you attach your code and results. *Note:* I would use a smooth wavefunction probably, but you may use a piecewise defined one if you deal with the kinetic energy as in the reading from Griffiths.

   (c) Use the numerical method of assignment 6, problem 3 to find the ground state energy to within one percent error. How close are your upper bounds from parts (a,b)? Attach your Maple code.

3. **Uniform Gravitational Field** *parts of Griffiths 8.5 and 8.6*

   Consider a ball of mass m that feels a uniform gravitational acceleration g in the $-x$ direction, as by the surface of the earth. Assume that the surface of the earth is at $x = 0$ and forms an infinite potential barrier.

   (a) First, write down what the potential energy is as a function of x.

   (b) Use the WKB approximation to find the allowed energies of the bouncing ball.