

Historical Introduction to Quantum Mechanics

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Where does classical physics start to fail?

• Blackbody (Thermal) Radiation

- What are we looking at?

• This is perfectly thermal radiation from an object or system that absorbs + emits light perfectly (no reflection)

+ To maintain equilibrium w/ surroundings, absorption + emission must be the same amount. (+ in each frequency range, too)

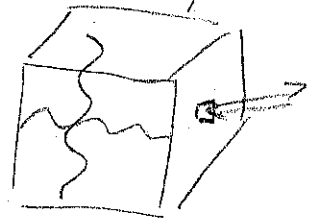
+ Real life "blackbodies" (approximate):

filament in light bulb, the sun (or star)

The most perfect one ever seen is (light from) the early universe

+ Model: Cubic box with reflecting walls

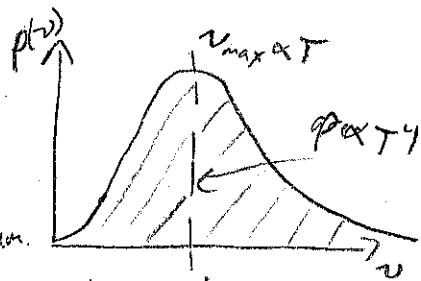
Light bounces inside. Absorbed or emitted from a small hole



• We want to understand the spectrum of the light. That's energy per frequency interval.

+ We will look at $p(\nu)$, the energy density spectrum.

$p(\nu)d\nu$ = energy density of light in freq. range ν to $\nu+d\nu$



+ Depends on temperature. Total energy density is

$$p = \int_0^{\infty} d\nu p(\nu) = \left(\frac{4\sigma}{c}\right) T^4 \quad \sigma - \text{St}$$

This is a version of the Stefan-Boltzmann law.

$$\sigma = 5.7 \times 10^{-8} \frac{\text{J}}{\text{m}^2 \text{s K}^4}$$

= Stefan-Boltzmann constant

+ The frequency of the peak is $\nu_{\text{max}} \propto \frac{1}{T}$.

This is Wien Displacement Law. Usually written $\lambda_{\text{max}} T = W$

$$W = 2.9 \times 10^{-3} \text{ mK}$$

- Two ingredients: "Density of States" and Energy/mode

• Density of states = # of different wave modes (wave #s) with same frequency per frequency interval per volume

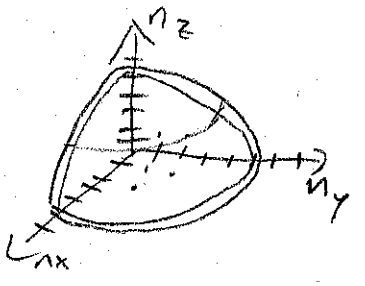
+ In the box, electromagnetic fields have Dirichlet b.c. (full treatment, see EM theory), specifically potential $\Phi = 0$ at box wall.

So $\Phi = A \sin(n_x \pi x/L) \sin(n_y \pi y/L) \sin(n_z \pi z/L)$, $n_x, n_y, n_z \in \text{integers}$

+ Wave vector is $\vec{k} = (\pi/L)(n_x \hat{x} + n_y \hat{y} + n_z \hat{z})$. Take all as positive

Therefore $2\pi r = (\pi/L)\sqrt{n_x^2 + n_y^2 + n_z^2} \equiv \frac{c\pi}{L} n$

+ The number of states in range ν to $\nu + d\nu$ is the number of integer lattice points in the spherical shell of appropriate radius.



This is approximately 1/8 volume of spherical shell.

$\frac{1}{8}(4\pi) r^2 dr = \frac{4\pi L^3}{c^3} \nu^2 d\nu$

+ Also, light waves have 2 polarizations, so total # of modes per volume is

$n(\nu) d\nu = (8\pi/c^3) \nu^2 d\nu$

• The average energy per mode:

+ At a fixed temperature, the probability of a given energy for a single mode follows the Boltzmann factor $P \propto e^{-E/kT}$ $k = \text{Boltzmann constant}$

+ Classically, each mode can have any energy given by the wave amplitude.

So $\langle E \rangle = \frac{\int_0^\infty dE E e^{-E/kT}}{\int_0^\infty dE e^{-E/kT}} = kT$

+ Classically, then, the energy density spectrum is

$\rho(\nu) d\nu = (kT) n(\nu) d\nu = \frac{8\pi kT}{c^3} \nu^2 d\nu$ Rayleigh-Jeans

This has the ultraviolet catastrophe: more and more energy density at larger frequencies

Planck's Law

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• Planck's guess about quantization: Each mode is allowed only certain energies $h\nu, 2h\nu, 3h\nu, \dots = nh\nu$ for positive integer n .

+ h is a constant of nature called Planck's constant w/ units of Energy \times time

+ The average energy in a mode of frequency ν :

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} nh\nu e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} e^{-nh\nu/kT}}$$

Well, $\sum_n e^{-nx} = \frac{1}{1-e^{-x}}$, $\sum_n ne^{-nx} = -\frac{d}{dx} \sum_n e^{-nx} = \frac{e^{-x}}{(1-e^{-x})^2}$

so

$$\langle E \rangle = h\nu / (e^{h\nu/kT} - 1) \leftarrow \begin{array}{l} \text{Drops when lowest energy } h\nu \\ \text{classical average.} \end{array}$$

+ Then the spectrum is Planck's Law

$$\rho(\nu) d\nu = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/kT} - 1} d\nu \quad \text{goes to 0 at } \nu \rightarrow \infty$$

• Does this fit what we wanted?

+ Wien's law comes from $d\rho/d\nu = 0$ to find max

$$3(e^{h\nu/kT} - 1) - \nu(h/kT e^{h\nu/kT}) = 0$$

Function only of $h\nu/kT \Rightarrow \nu_{\max} \approx (2.8h/k)T$

+ Stefan-Boltzmann comes from integral

$$\rho = \frac{8\pi h}{c^3} \int_0^{\infty} \frac{\nu^3}{e^{h\nu/kT} - 1} d\nu = \frac{8\pi k^4 T^4}{c^3 h^3} \int_0^{\infty} \frac{x^3}{e^x - 1} dx = \left(\frac{8\pi^5 k^4}{15 h^3} \right) T^4$$

+ Matching to Wien + Stefan-Boltzmann constants give

$$h = 6.626 \times 10^{-34} \text{ J s} \quad k = 1.381 \times 10^{-23} \text{ J/K}$$

• Today, we interpret this as meaning light comes in discrete clumps called photons

• Photoelectric Effect

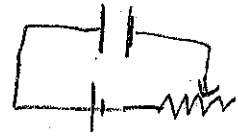
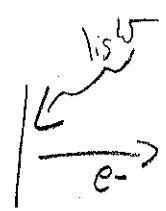
- Experiment: Light on metal

• light shining on metal can eject electrons.

+ You can measure the kinetic energy of the electrons if the plate is in a capacitor

+ Current flow stops when $eV = \text{max KE}$

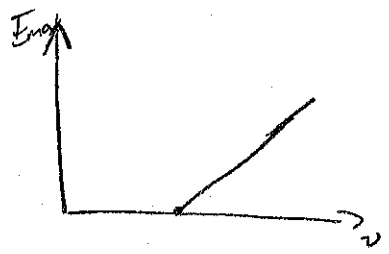
This stops ejected e^- before they hit the other plate



• Results: Dependent on frequency of light

+ At low frequency, there is no current.

+ At a critical ν , current turns on, and $E_{\text{max}} = h\nu - E_0$



- Interpretation:

• Classically, this is baffling.

+ Energy in a wave depends on amplitude, not frequency. So E_{max} should not depend on frequency

+ You should be able to eject electrons at arbitrarily low freq. if you have a strong enough light source.

• Einstein's Quantum interpretation: extend Planck's idea

+ Light comes in individual particles, and electrons interact pretty much just with one at a time \leftarrow photons

+ The energy of a photon is $E = h\nu$. When e^- absorbs a photon, it's KE is $h\nu - \text{binding energy} \leftarrow E_0$

+ By the way, that's why cell phones don't cause brain cancer

+ The Compton effect leads to the same ideas.

+ But light is also a wave: wave-particle duality

Bohr Atomic Model

- Problems with the atom

• A hot atomic gas does not emit a continuous spectrum (i.e., it does not emit thermal light).

+ Radiation comes out as discrete spectral lines

+ for H, $1/\lambda = R \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$, $R = 1.097 \times 10^7 \text{ m}^{-1} = \text{Rydberg constant}$

• An e^- orbiting a nucleus accelerates. Accelerating charges radiate.

The electron should rapidly fall into the nucleus.

- The Bohr model:

• Assume angular momentum is quantized in units of $\hbar = h/2\pi$.

+ This says $pr = n\hbar$, $n = 1, 2, 3, \dots$, or $2\pi r = n(\lambda/p)$

Suggests an integer # of wavelengths $\lambda = h/p$

+ $\lambda = h/p$ is the de Broglie wavelength. Means that matter has wave + particle nature

• Use classical mechanics

+ Force balance

$$\frac{e^2}{4\pi\epsilon_0} \frac{1}{r^2} = \frac{mv^2}{r} = \frac{(pr)^2}{mr^3} = \frac{n^2 \hbar^2}{mr^3}$$

$$\Rightarrow r = \frac{4\pi\epsilon_0 \hbar^2}{me^2} n^2, \quad v = \frac{e^2}{4\pi\epsilon_0 \hbar} \frac{1}{n}, \quad n = 1, 2, 3, \dots$$

+ Total energy

$$E_n = \frac{1}{2} mv^2 - \frac{e^2}{4\pi\epsilon_0} \frac{1}{r} = -\frac{m}{2\hbar} \left(\frac{e^2}{4\pi\epsilon_0} \right)^2 \frac{1}{n^2} = (-13.6 \text{ eV}) \frac{1}{n^2}$$

+ The spectral line wavelengths come from

$$E(\text{photon}) = hc/\lambda = E_m - E_n$$

• Bohr model is ultimately incorrect but introduces key ideas: quantization of angular momentum + energy and particle-wave duality of matter.