

PHYS-3301 Homework 9 Due 14 Nov 2012

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. 4-Velocity and Position

Consider a spaceship flying along the x axis. The spatial part of its 4-velocity relative to a space station is $\vec{U} = c \sinh(\omega\tau)\hat{x}$ as a function of its proper time τ . *Hint:* In this problem, you will want to know the following hyperbolic trig identities:

$$\cosh^2 \theta - \sinh^2 \theta = 1, \quad \frac{d \cosh \theta}{d\theta} = \sinh \theta, \quad \frac{d \sinh \theta}{d\theta} = \cosh \theta.$$

Remember that you can use the first of these to evaluate quantities like $\cosh(\sinh^{-1} z)$, etc.

- Find the time component U^0 of the 4-velocity and use it to show that the relativistic gamma factor can be written as $\gamma = \sqrt{1 + \vec{U}^2/c^2}$.
- Find the spaceship's position x and the coordinate time t as a function of the ship's proper time τ . Choose $t = 0$ at $\tau = 0$.
- Write the (x component of the) spaceship's coordinate velocity $u = dx/dt$ and its position x as a function of coordinate time t .

2. Energy Release Efficiencies

In this question, you will find the fraction of mass energy released in a couple of nuclear reactions. It will be useful for you to know that one atomic mass unit (aka one *dalton*) is $930 \text{ MeV}/c^2$ and also $1.7 \times 10^{-27} \text{ kg}$ (both to two significant figures).

- In a nuclear reactor, a slow (nonrelativistic) neutron of mass $940 \text{ MeV}/c^2$ hits a Uranium-235 nucleus (mass 240 dalton), which breaks into various fragments. This *fission* (splitting) reaction releases 200 MeV of energy into heat, which the reactor can then convert to work. What fraction of the initial mass energy is converted to useful heat?
- The sun produces heat through the *fusion* of light nuclei into heavier ones. Most of this "hydrogen burning" effectively converts 4 protons (mass $940 \text{ MeV}/c^2$) into a He-4 nucleus (mass 4.0 dalton), 2 positrons (aka anti-electrons) of mass $0.51 \text{ MeV}/c^2$, two neutrinos (essentially zero mass), and photons (zero mass). The two positrons immediately find two electrons and annihilate into photons. Assume that all of the energy above the rest energy of the final state particles is converted to heat (we ignore the few percent of energy that is carried away by the neutrinos). What fraction of the initial mass energy is converted to heat?

3. SN1987A and Neutrino Masses

On 23 Feb 1987, astronomers were startled by the observation of a new supernova in the Large Magellanic Cloud, a satellite galaxy of our Milky Way. However, the first observation of this supernova was several hours earlier by the detection of neutrinos, which was confirmed by two detectors. (The neutrinos arrived before the light because light is trapped for a while by all the matter inside the exploding star.) The fact that the neutrinos all arrived within a few seconds of each other after traveling for more than 100,000 lightyears allows us to put tight constraints on the mass of the neutrino. This problem will guide you through a real calculation of this limit.

- (a) Show that a neutrino with energy $E \gg mc^2$ has a speed approximately given by

$$\frac{|\vec{u}|}{c} \approx 1 - \frac{1}{2} \left(\frac{mc^2}{E} \right)^2 . \quad (1)$$

Hint: We gave formulas in class for energy both in terms of the spatial momentum and in terms of the speed. Try looking at those. Then you will need to make an expansion in powers of mc^2/E .

- (b) Light (once free of the matter in the supernova) takes a time $t_0 = 5.3 \times 10^{12}$ s to travel from SN1987A to the earth. How long would a neutrino of energy E take to reach earth from the supernova? Work to the lowest non-trivial order in mc^2/E and give the answer in terms of t_0 , m , c , and E .
- (c) The Kamioka detector in Japan detected several neutrinos. The first arrived with energy 21.3 MeV, and another with energy 8.9 MeV arrived 0.303 s later. Assuming that the second neutrino left the supernova no more than 1 s before the first, what is the maximum neutrino mass m ? For simplicity, we are ignoring the possible error in the measurements. *Hint:* The observation time of each neutrino is its emission time plus its travel time; take the difference of these and be careful of signs.