PHYS-3301 Homework 6 Due 24 Oct 2012

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Twins Again** *Inspired by Barton 5.6*

   Remember our twins Frankie and Fannie. Frankie stays on earth, while Fannie flies off (at birth) to a star 20 lightyears away at constant speed $u = 4c/5$. When she reaches the star, Fannie turns around immediately and returns to the earth at the same speed. This problem will start to help you see why Frankie and Fannie are different. *Hint:* It will probably help you to look at the solution set for assignment 4.

   (a) The earth and the star are at positions $x = 0$ lightyears and $x = 20$ lightyears in the earth's inertial reference frame. Write Fannie's path as a function $x(t)$, where t is measured in the earth's reference frame.

   (b) Every ten years on his birthday, Frankie sends a light signal to Fannie. Does Fannie receive any of the signals before she reaches the star and turns around?

   (c) Draw a spacetime diagram in the earth's reference frame. Show Fannie's worldline and the worldlines of the light signals.

   Note: Light speed can be written in appropriate units as $c = 1$ lightyear/year. Those units might make life easier.

2. **A Moving Object** *Barton 6.6 plus*

   A relativistic car moves along the x axis, passing $x = 0$ at $t = 0$. Its velocity is $u(t) = dx/dt = c/\cosh(\omega t)$. **Important:** In the following, you will find some integrals involving hyperbolic trig functions. You may use computer programs such as Maple or Mathematica to do them ***only*** if you *cite the program you use* and *check the result of the indefinite integral by differentiating it and seeing that you get the integrand.*

   (a) Sketch the worldline of the car on a spacetime diagram. Can a real car follow this path exactly?

   (b) Find the relation between the time t and the car's proper time τ. Choose integration constants so that $\tau = 0$ when $t = 0$. Be careful taking signs if you take a square root.

   (c) Find u as a function of τ and then find $dx/d\tau$. *Hint:* Remember that $u = dx/dt$, not $dx/d\tau$.

   (d) Since the car moves, it doesn't experience as much proper time as coordinate time t. Find the total time lag Δ over all time, which is defined as

   $$\Delta = \lim_{t\to\infty} [t - \tau(t)] - \lim_{t\to-\infty} [t - \tau(t)] \ .$$

3. **Distribution of Photons from Moving Pions** *Barton 4.10 expanded*

   Neutral pions (aka π^0 mesons) are subatomic particles that decay into photons (light particles). In the rest frame S' of the pions, the photons are emitted uniformly in angle ($f'(\theta', \phi') = 1/4\pi$). If the pions move at speed v along the $+x$ axis in the lab frame S, more photons will be emitted toward positive x than $-x$ in the lab frame. In this problem, we'll calculate how many more using the following logic. *Hint:* you might also want to look at assignment 2, problem 2.

(a) Consider a photon emitted from a lump of pions moving at speed c at an angle θ' to the x' axis as measured in the S' frame. Show that its velocity makes an angle θ with respect to the x axis as measured in the S frame, where

$$\cos\theta = \frac{\cos\theta' + v/c}{1 + v\cos\theta'/c} \ . \tag{1}$$

(b) The dividing line between forward emission and backward emission in the lab frame S is the equator $\cos\theta = 0$. Find the corresponding angle θ'_0 in the S' frame. Remember, we're talking about photons which move at speed c.

(c) Finally, integrate the angular distribution of photons over solid angles with $\theta' < \theta'_0$ and subtract the integral over solid angles with $\theta' > \theta'_0$ to get the difference between forward and backward emission as measured from the lab frame.