PHYS-3301 Homework 2 Due 26 Sept 2012

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Velocity Transformations in Spherical Coordinates**

   In this problem, the frame S' moves at velocity $\vec{v} = v\hat{z}$ relative to frame S (the axes of the two frames are aligned, and the origins coincide at time zero).

   (a) In frame S, a particle moves with velocity $\vec{u}$ in the $y - z$ plane. This velocity makes an angle θ with the z axis, where $\tan\theta = |u_y|/u_z$. Find the magnitude of the velocity $\vec{u}'$ of the particle in the S frame in terms of $|\vec{u}|$, v, and $\cos\theta$.

   (b) Find $\tan\theta'$ in terms of $\vec{u}, \vec{v}$, where θ' is angle the particle's velocity makes with the z axis in the S' frame.

   (c) Show that

   $$\cos\theta' = \frac{\cos\theta - v/|\vec{u}|}{\sqrt{1 - 2v\cos\theta/|\vec{u}| + v^2/|\vec{u}|^2}} \approx \cos\theta - \frac{v}{|\vec{u}|}\sin^2\theta \ , \tag{1}$$

   where the approximation is valid if $v/|\vec{u}| \ll 1$. *Hint:* To show the final approximation, you need to Taylor expand the exact expression in terms of the variable $x = v/|\vec{u}|$ and keep ony terms to first order. This process can be simplified if you replace the denominator using the binomial expansion $(1 + a)^n \approx 1 + na$ for $a \ll 1$ and n any power.

2. **Transformed Angular Distribution** *Barton 2.5 elaborated*

   In this problem, consider a lump of radioactive material emitting a bunch of alpha particles. In the rest frame S' of the material, the alpha particles all have speed u', and their velocities have a uniform angular distribution $f'(\theta', \phi') = N/4\pi$. θ' and ϕ' are the usual polar and azimuthal angles defined in the S' frame.

   Suppose we want to change frames to a laboratory frame S. (As in problem 1, S' moves at speed v along the z axis relative to S.) Find the *asymmetry* σ in the S frame angular distribution, the number of particles emitted toward positive z minus the number emitted toward negative z in frame S. Use the following steps:

   (a) Since nothing depends on the azimuthal angle, write down the reduced angular distribution $f'(\theta')$ in the S' frame.

   (b) The particles emitted toward positive z in S have $\cos\theta > 0$ (and $\cos\theta < 0$ if toward negative z). Find $\cos\theta'$ when $\cos\theta = 0$ and call the corresponding angle θ'_0.

   (c) Therefore, the number emitted forward in frame S is the number emitted in frame S' with $\theta' < \theta'_0$, etc. Calculate

   $$\sigma = 2\pi\left[\left(\int_0^{\theta'_0} d\theta' \sin\theta' f'(\theta')\right) - \left(\int_{\theta'_0}^{\pi} d\theta' \sin\theta' f'(\theta')\right)\right] \ . \tag{2}$$