PHYS-3301 Homework 1 Due 19 Sept 2012

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Choosing Frames Wisely**

   In both parts, clearly state what inertial reference frame you use to solve the problem.

   (a) *Barton 2.2 rephrased* A river flows at 5 km/hr, and a boat in it can move 8 km/hr relative to the water. As the boat moves upstream, the driver hears a splash but only realizes that it was the life preserver falling overboard 15 minutes later. The driver turns around and heads to retrieve the life preserver. How soon can the boat catch up to the life preserver?

   (b) *Barton 2.10 rephrased* A cannonball is launched in an arc with velocity $\vec{u}$. At the top of its trajectory, a chemical charge in it explodes into two parts of masses m_1 and m_2 that separate in the horizontal direction only. The explosion releases energy E, which essentially all goes into the kinetic energy of the cannonball pieces. Show that they are separated by a distance $u_y/g\sqrt{2E(m_1+m_2)/m_1 m_2}$ when they land, where u_y is the initial vertical component of the velocity.

2. **CM Frame**

   Consider n particles (labeled $i = 1, \ldots n$) in motion and define the total mass $M = \sum_i m_i$ and center of mass velocity $\vec{U} = (1/M)\sum_i m_i \vec{u}_i$.

   (a) First, show that the difference between two velocities is invariant under boost transformations. Then write the CM frame velocity $\vec{u}'_i$ of each particle as the difference of two velocities in our given lab frame.

   (b) Show that the total kinetic energy can be written as $K_{tot} = \vec{P}^2/2M + K_{int}$, where $\vec{P} = M\vec{U}$ is the total momentum and K_{int} is the kinetic energy measured in the CM frame. Write K_{int} solely in terms of quantities that are invariant under Galilean transformations, which proves that it is invariant.

3. **Angular Momentum** *Barton 2.9 plus*

   Angular momentum for a single particle is defined as $\vec{L} = \vec{x} \times \vec{p}$ in a given reference frame S.

   (a) Show that the value $\vec{L}'$ of the angular momentum measured in a reference frame S' is

      i. $\vec{L}' = \vec{L} - \vec{b} \times \vec{p}$ if S' is translated by $\vec{b}$ compared to S. (Meaning that angular momentum does depend on your choice of origin.)

      ii. $\vec{L}' = \vec{L} + \vec{v} \times (m\vec{x} - \vec{p}t)$ if S' is boosted by $\vec{v}$ compared to S.

   (b) Define the total angular momentum $\vec{\mathbf{L}} = \sum_i \vec{L}_i$ as the sum of the individual particle angular momenta in a system. Suppose $\vec{\mathbf{L}}$ is measured in the CM frame of the system with the origin chosen at the position of the center of mass. Show that the total angular momentum $\vec{\mathbf{L}}'$ measured in any frame S' that is translated and boosted with respect to S is equal to $\vec{\mathbf{L}}$.