QFT Homework 1 Due 17 Sept 2012

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF (prepared with LaTeX) to Dr. Frey.

Reading: Tong's notes chapter 1. Also, read as much of Srednicki chapters 1 & 2 as possible.

To Be Marked:

1. Noether's Theorem From Lahiri & Pal 2.9

Consider a complex scalar field with Lagrangian density

$$\mathcal{L} = -\partial_{\mu}\bar{\phi}\partial^{\mu}\phi - V\left(|\phi|^{2}\right) , \qquad (1)$$

where $\bar{\phi}$ is the conjugate of ϕ . This Lagrangian is invariant under the transformation $\phi \rightarrow \exp[-iq\theta]\phi$ and its conjugate.

- (a) Find the equation of motion for ϕ from the given Lagrangian (remember to treat ϕ and $\overline{\phi}$ as independent variables).
- (b) Use Noether's theorem to find the corresponding conserved current j^{μ} and verify that $\partial_{\mu}j^{\mu} = 0.$
- (c) If we include the electromagnetic field, the Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \left[(\partial - iqA)_{\mu}\bar{\phi}\right]\left[(\partial + iqA)^{\mu}\phi\right] - V\left(|\phi|^2\right) , \qquad (2)$$

where $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$. Find the conserved current j^{μ} for this Lagrangian.

(d) Find the equation of motion for A_{μ} and show that it takes the form $\partial_{\mu}F^{\mu\nu} = j^{\nu}$, where j^{μ} is the same current you found in part (c).

Useful Excercises To Do On Your Own (not to be marked):

2. Filling In Blanks Srednicki problems 2.1 to 2.6