

QFT Take-Home Exam Due 14 Dec 2012 5PM

Must be handed in to Dr Frey or to the dropbox outside 2L26 by 5PM on Fri 14 Dec 2012. If you wish to turn it in ahead of time, you may email a PDF (prepared with LaTeX) to Dr. Frey.

You may consult the Srednicki text, your class notes, your homework assignments, or the posted homework solutions. No other references are allowed, and you may not collaborate.

1. A Theory with Two Couplings

Consider the general Lagrangian density of a single scalar field ϕ

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \mathcal{L}_{ct}, \quad V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{g}{3!}\phi^3 + \frac{\lambda}{4!}\phi^4, \quad (1)$$

where \mathcal{L}_{ct} contains counterterms. Assume that g^2/m^2 is of the same order of magnitude as λ .

- (a) [10 points] Find the tree-level amplitude for $\phi\phi \rightarrow \phi\phi$ scattering as a function of the Mandelstam variables s, t, u . In the $s \rightarrow \infty$ limit, argue that the ϕ^4 interaction dominates the amplitude (except for forward or backward scattering) and find the approximate cross section in that limit. Work in 4D in this part of the problem.
- (b) [10 points] Write down the Lagrangian in terms of bare field and parameters. Define the conversion between the bare and renormalized field and parameters and find the relation between those conversions and the counterterms.
- (c) [10 points] Suppose you are given that $\lambda = 0$. Argue that this should be a super-renormalizable theory in 4D. Draw all the primitively divergent Feynman diagrams in that case and write down all the necessary counterterms. Do you need a counterterm for a ϕ^4 interaction, and is it consistent to take $\lambda = 0$?
- (d) [20 points] Now take all the couplings to be nonzero. Work in the MS scheme in $d = 4 - \epsilon$ dimensions and find all necessary counterterms in terms of the mass and couplings at one-loop order. *NOTE:* You do not have to evaluate loop integrals in detail, just the divergent parts! Use dimensional regularization.
- (e) [25 points] Using the fact that the bare field and parameters are independent of renormalization scale, find the anomalous dimension of ϕ , the anomalous dimension of the mass, and the beta functions of g and λ .