

(Addendum on degenerate perturbation theory)

(Addendum on degenerate perturbation theory)

(59a)

• Diagonalizing W may be easier if there is another hermitian operator \mathcal{O} that commutes with both H_0 and H_1

+ So we can write 0^{th} order stationary states as \mathcal{O} eigenstates

+ Also, $W_{mn} = 0$ unless $|\psi_m^0\rangle$ and $|\psi_n^0\rangle$ have same \mathcal{O} eigenvalue

Proof

$$W_{mn} = \langle \psi_m^0 | H_1 | \psi_n^0 \rangle = \frac{1}{\lambda_n} \langle \psi_m^0 | H_1 \mathcal{O} | \psi_n^0 \rangle$$

where $\lambda_n = \text{eigenvalue of } |\psi_n^0\rangle$

But then \mathcal{O} commutes

$$\begin{aligned} W_{mn} &= \frac{1}{\lambda_n} \langle \psi_m^0 | \mathcal{O} H_1 | \psi_n^0 \rangle = \frac{1}{\lambda_n} \langle \psi_n^0 | H_1 \mathcal{O} | \psi_m^0 \rangle \\ &= (\lambda_m / \lambda_n) \langle \psi_m^0 | H_1 | \psi_n^0 \rangle = (\lambda_m / \lambda_n) W_{mn} \end{aligned}$$

This is zero unless $\lambda_m = \lambda_n$

+ So look for some conserved hermitian operator.

On HW, this is L_z