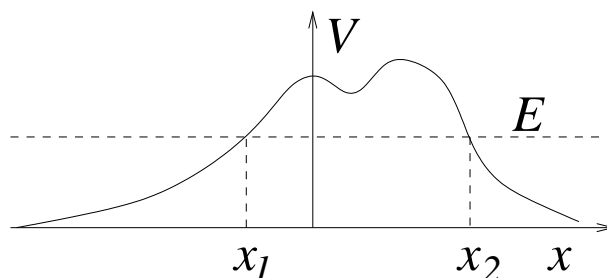


## PHYS-4601 Homework 20 Due 22 Mar 2012

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

### 1. Tunneling in the WKB Approximation

Consider a potential  $V(x)$  in 1D as in the following figure (don't try to find the actual functional form of  $V$ ) and a stationary scattering state with energy  $E$  as indicated by the dashed line.



The classical turning points are  $x_1$  and  $x_2$ . In the region to the left, where  $E > V(x)$  and  $x < x_1$ , we can write the wavefunction in the WKB approximation as

$$\psi(x) = \frac{A}{\sqrt{p(x)}} \exp\left[-i \int_x^{x_1} dx' p/\hbar\right] + \frac{B}{\sqrt{p(x)}} \exp\left[i \int_x^{x_1} dx' p/\hbar\right], \quad p(x) = \sqrt{2m(E - V(x))}, \quad x < x_1, \quad (1)$$

where  $A$  is the coefficient of the incident wave and  $B$  the coefficient of the reflected wave. Between the turning points,

$$\psi(x) = \frac{C}{\sqrt{\rho(x)}} \exp\left[-\int_{x_1}^x dx' \rho/\hbar\right] + \frac{D}{\sqrt{\rho(x)}} \exp\left[\int_{x_1}^x dx' \rho/\hbar\right], \quad \rho(x) = \sqrt{2m(V(x) - E)}, \quad x_1 < x < x_2. \quad (2)$$

And finally, off to the right,

$$\psi(x) = \frac{F}{\sqrt{p(x)}} \exp\left[i \int_{x_2}^x dx' p/\hbar\right], \quad x > x_2 \quad (3)$$

is the transmitted wave.

(a) Last semester, on assignment 7, you saw that the transmission coefficient is written as

$$T = \frac{|\psi_{trans}|^2 p_{trans}}{|\psi_{inc}|^2 p_{inc}}, \quad (4)$$

where  $\psi_{trans}$  and  $\psi_{inc}$  are the transmitted and incident parts of the wavefunction and  $p_{trans}$ ,  $p_{inc}$  are the momenta in the regions  $x > x_2$  and  $x < x_1$  respectively. Find  $T$  in terms of the coefficients  $A, B, C, D, F$ .

(b) Rewrite the connection formulas given in the class notes in terms of complex exponentials rather than sines and cosines. *Hint*: Note that the coefficients you use in the connection formulas are not necessarily those given in the wavefunction above.

- (c) Use your new connection formulas as given in part (b) to write the coefficients  $C$ ,  $D$  in terms of  $F$  and then  $A$ ,  $B$  in terms of  $F$ . You should find that

$$A = \frac{1}{2} \left( 2\theta + \frac{1}{2\theta} \right) F \text{ where } \theta = \exp \left[ \int_{x_1}^{x_2} dx' \rho(x')/\hbar \right]. \quad (5)$$

*Hint:* As we did in class when studying bound states, you will find that the connection formulas give two different forms for the wavefunction in the region  $x_1 < x < x_2$ . You will need to relate those.

- (d) Using your results from parts (a,c), find the transmission coefficient. In the limit that the barrier is high and wide, show that

$$T \approx \exp \left[ -2 \int_{x_1}^{x_2} dx' \rho(x')/\hbar \right], \quad (6)$$

as we approximated in class.