PHYS-4601 Homework 19 Due 15 Mar 2012

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Hydrogen with the Yukawa Potential** *Griffiths 7.14 refined*

   If the photon had a mass, the Coulomb potential would be changed to the *Yukawa potential*

   $$V(\vec{x}) = -\frac{e^2}{4\pi\epsilon_0}\frac{e^{-\mu r}}{r} \ , \tag{1}$$

   where μ is proportional to the photon mass. Effectively, the photon mass screens out some of the effect of the proton charge on the electron. Use a trial wavefunction

   $$\psi(\vec{x}) = Ce^{-Zr/a}Y_0^0(\theta,\phi) \tag{2}$$

   (based on the hydrogen ground state wavefunction, where C is a normalization constant, Z is a free parameter representing the charge screening, and a is the Bohr radius), and estimate the ground state energy of the Yukawa potential with the variational principle. Follow the following steps:

   (a) Find the normalization constant C as a function of the parameter Z.

   (b) For a fixed value of Z, calculate $\langle H\rangle$. Assume $\mu a \ll 1$ and keep only terms up to order $(\mu a)^2$. *Hint*: To evaluate the expectation value of the kinetic energy, you may find it helpful to use

   $$p^2\cdot\psi = -\hbar^2\nabla^2\psi = -\frac{\hbar^2}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\vec{L}^2\cdot\psi \ . \tag{3}$$

   There is also a helpful integral identity in the back cover of Griffiths.

   (c) Minimize your answer from part (b) with respect to Z to estimate the ground state energy.

2. **Anharmonic Oscillator Again**

   In assignment 17 problem 4, you considered a particle moving in the 1D potential

   $$V(x) = \frac{1}{2}m\omega^2x^2 + gx^3 \tag{4}$$

   and considered perturbations of the harmonic oscillator ground state $|0\rangle$. Using the variational method, show that the true ground state energy of this potential is unbounded below (that is, if I give you any real number, demonstrate that the ground state energy is less than that number). We say that this potential is unstable and has no ground state. *Hint*: Think about a simple trial wavefunction that approximates a delta function in position.

3. **Uniform Gravitational Field** *parts of Griffiths 8.5 and 8.6*

   Consider a ball of mass m that feels a uniform gravitational acceleration g in the $-x$ direction, as by the surface of the earth. Assume that the surface of the earth is at $x=0$ and forms an infinite potential barrier.

   (a) First, write down what the potential energy is as a function of x.

   (b) Use the WKB approximation to find the allowed energies of the bouncing ball.