PHYS-4601 Homework 14 Due 26 Jan 2012

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **3-Particle States** *a mix of Griffiths 5.7 and 5.33*

   Consider three particles, each of which is in one of the single-particle states $|\alpha\rangle$, $|\beta\rangle$, or $|\gamma\rangle$, which are orthonormal.

   (a) If the particles are bosons, write down the state where one particle is in each of $|\alpha\rangle$, $|\beta\rangle$, and $|\gamma\rangle$. *Hint*: This state must be symmetric under the exchange of *any* pair of the bosons.

   (b) How many linearly independent states can you form if the particles are fermions? Write down all the possible linearly independent states. *Hint*: Similarly to the above, these states must be antisymmetric under the exhange of *any* pair of the fermions.

2. **Estimating Helium Better** *Griffiths 5.11 clarified*

   In this problem, we will estimate the ground state energy of a helium atom. We will imagine that the electron repulsion is a correction to the attraction between the electrons and the nucleus.

   (a) Consider the states of a single electron around a helium nucleus (which has twice the charge of a proton). Argue that the "helium Bohr radius" $a_{\mathrm{He}} = a/2$, where a is the usual Bohr radius, and that therefore the single-electron ground state wavefunction is given by

   $$\langle\vec{x}|n=1,\ell=0,m=0\rangle = \sqrt{\frac{8}{\pi a^3}}e^{-2r/a} \ . \tag{1}$$

   Next assume that the two electron helium groundstate is $|n=1,\ell=0,m=0\rangle_1|n=1,\ell=0,m=0\rangle_2|s=0,m_s=0\rangle$, where the total spin state is the antisymmetric singlet. (The spatial wavefunction is given by Griffiths eqn [5.30].) Briefly argue that the energy of this state, in the absence of electron repulsion, is given by Griffiths eqn [5.31].

   (b) Now find $\langle|\vec{x}_1-\vec{x}_2|^{-1}\rangle$ in this state, as follows:

      1. Use the trick of setting the z axis for $\vec{x}_2$ along $\vec{x}_1$ and the law of cosines to see $|\vec{x}_1-\vec{x}_2| = \sqrt{r_1^2+r_2^2-2r_1r_2\cos\theta_2}$.
      2. Do the angular integrals for $\vec{x}_2$, noting that

      $$\int_0^\pi d\theta \sin\theta f(\cos\theta) = \int_{-1}^1 dx f(x) \ .$$

      Your result will have square roots of perfect squares, which are equal to absolute values. *Be careful of that!*
      3. Carry out the r_2 integral in two parts, $0 < r_2 \le r_1$ and $r_1 < r_2 < \infty$.
      4. Now do the $\vec{x}_1$ integrals.

   Hint: The "exponential integrals" formula in the back cover of Griffiths will be helpful.

   (c) Estimate the change in the ground state energy due to the electron repulsion as

   $$\Delta E = \frac{e^2}{4\pi\epsilon_0}\left\langle\frac{1}{|\vec{x}_1-\vec{x}_2|}\right\rangle \ . \tag{2}$$

   Write ΔE in terms of the Bohr radius a and estimate its value in eV. Then add this to the energy from part (a) to get a rough estimate of the He ground state energy. *Hint*: Remember that the hydrogen ground state energy is $-\hbar^2/2ma^2 = -13.6$ eV.

3. **Finishing Bell's Inequality** *Griffiths 4.50*

   Consider 2 distinguishable spin 1/2 particles in the singlet ($s = 0$) total spin state. If $\vec{a}$ and $\vec{b}$ are two unit vectors, show that

$$\left\langle \left(\vec{a} \cdot \vec{S}^{(1)}\right) \left(\vec{b} \cdot \vec{S}^{(2)}\right) \right\rangle = -\frac{\hbar^2}{4} \vec{a} \cdot \vec{b} \ . \tag{3}$$

   Hint: Think about a convenient choice of axes.