PHYS-4601 Homework 13 Due 19 Jan 2012

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Hydrogen Ground State**

   In this problem, the wavefunction is always that of the ground state $n = 0$, $\ell = 0$, $m = 0$ of the hydrogen atom. Give all answers in terms of pure numbers and the Bohr radius.

   (a) *Griffiths 4.14* For what value of r are you most likely to find the electron between r and $r + dr$? Recall that the volume of each spherical shell changes with radius.

   The next 2 parts are based on Griffiths 4.13

   (b) Find $\langle x\rangle$, $\langle y\rangle$, and $\langle z\rangle$. Do not do any integration but rather argue based on the rotational symmetry of the wavefunction.

   (c) Find $\langle r^2\rangle$. Using your result and the rotational symmetry of the wavefunction, find $\langle z^2\rangle$ (no additional integration allowed).

2. **High Angular Momentum** *from Griffiths 4.46*

   In this problem, consider states of the hydrogen atom with principle quantum number n and maximum orbital angular momentum $\ell = n - 1$.

   (a) Use the recursion relation for the series solution to the radial equation to show that the radial wavefunction is

   $$R(r) = Ar^{n-1}e^{-r/na} \ , \tag{1}$$

   where A is a normalization constant, and then find A.

   (b) Find $\langle r\rangle$ and $\langle r^2\rangle$.

   (c) Find the uncertainty σ_r and show that $\sigma_r/\langle r\rangle \propto 1/\sqrt{n}$ for large n. How does this calculation relate to the "quantum earth" problem on the last assignment?

3. **Multiple Particle Wavefunctions** *based on a problem in Ohanian*

   Consider two free spin 1/2 particles, which have single particle states $|\psi_1\rangle = |\vec{p}_1\rangle|+\rangle$ and $|\psi_2\rangle = |\vec{p}_2\rangle|-\rangle$. These states are factorized into spatial states (in this case, momentum eigenstates) and spin states (eigenstates of S_z).

   (a) Write the two-particle wavefunction if the two particles are distinguishable (say, particle 1 is a proton and particle 2 is an electron).

   (b) Now suppose that both particles are electrons, so they are indistinguishable. Write the two-particle state which is an eigenfunction of the total spin operator $\vec{S}^2$ with eigenvalue given by $s = 0$.

   (c) Keeping indistinguishable electrons, now write the two-particle wavefunction for with total spin eigenvalues $s = 1$, $m = 0$.

   (d) Finally, consider the case where the particles are indistinguishable but instead have spin 0, so there is no spin part of their states. Write the allowed two-particle state.