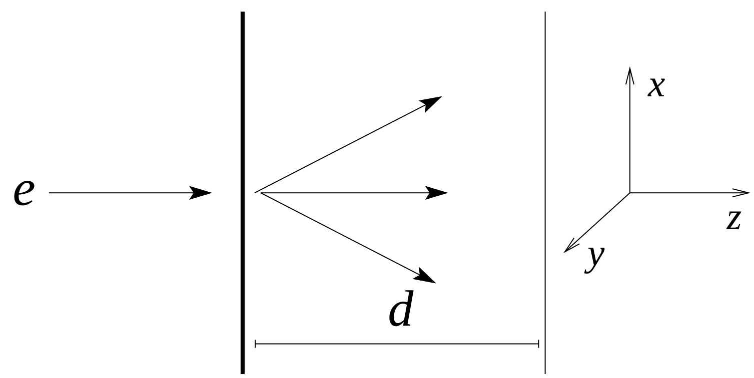
PHYS-4601 Homework 1 Due 15 Sept 2011

This homework is due in class on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Double Slit Experiment**

   Imagine an electron moving from the left toward a barrier with a small slit or slits. Its wavefunction must vanish on the barrier, but it can scatter through any slits into the region on the right. At the far right (a distance d from the barrier) is a screen that measures the location of the impact of any electron, as in the figure below. The orientation of the coordinate axes is shown in the figure.



   (a) The wavefunction of an electron of fixed energy that has passed through a single slit is approximately e^{ikr}/r far to the right, where r is the distance from the slit to the point in question and k is some constant. If there is a single slit at $x = y = 0$, what is the probability density for detecting an electron at location x, y on the screen? Make a sketch of the probability density as a function of x for $y = 0$.

   (b) Now consider a case with two small slits at $y = 0$, $x = \pm a$. Since the Schrödinger equation is linear, you can superpose the wavefunctions of electrons passing through the individual slits to find the wavefunction for an electron passing through both slits simultaneously. What is the probability density at location x, y on the screen? Sketch this as a function of x for $y = 0$.

   (c) Imagine that electrons have an extra "quantum number" attached to their wavefunction, so electrons with different quantum numbers are distinct particles. Now suppose that only electrons with a specific value of the quantum number can pass through the top slit, while only electrons with another value can pass through the bottom slit. Explain qualitatively what the probability distribution for finding an electron on the screen would look like in this case.

2. **Eigenstates of the Position Operator**

   In this problem, consider quantum mechanics in one dimension.

   (a) What is the eigenfunction of the position operator x with eigenvalue x_0 (that is, what Ψ satisfies $x\Psi = x_0\Psi$ with x and operator and x_0 a number)?

   (b) Explain how you would calculate the variance in the expectation of the momentum operator p in the eigenfunction you found in part a. What does the answer mean physically?

Hint: You might find the following formula helpful:

$$\delta(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikz} dk \ .$$

3. **Time Evolution of a Wavefunction**

   A particle of mass m is confined to the interval $0 < x < L$ in one dimension, and it has wavefunction

   $$\Psi(x,t) = Ae^{-i\alpha t} \sin\left(\frac{\pi x}{L}\right) \left[1 + e^{-i\beta t} \cos\left(\frac{\pi x}{L}\right)\right] \ .$$

   (a) What is functional form of the potential energy $V(x)$ in the region $0 < x < L$?

   (b) Determine the value of A and β. Is it possible to determine the value of α without more information, and does it matter?

   (c) Find the expectation value of the momentum of the particle as a function of time. Explain your results qualitatively by sketching the wavefunction at a few different times.

4. **Homework Comments**

   The following questions are **ungraded**, but your answers are greatly appreciated.

   (a) On a scale of 1 to 10, with 1 being very easy, 10 very difficult, and 5 the average of homeworks from your physics classes last year, how difficult was this assignment?

   (b) On a scale of 1 to 10, with 1 being very short, 10 very long, and 5 the average of homeworks from your physics classes last year, how long was this assignment?