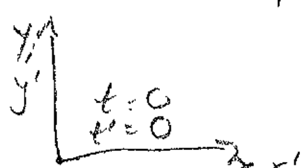
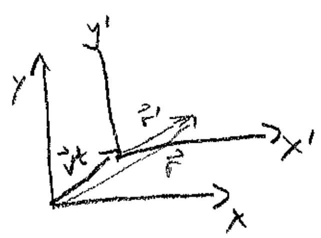
Galilean (aka Newtonian) Relativity

low speeds commonly called "nonrelativistic," but that's not really true

- Relativity Principle again
  - All laws of nature have same mathematical form in all inertial frames

  or
  - No expt performed in one inertial frame can determine motion relative to another inertial frame
- This tells us certain transformations are symmetries.
  Which ones? Uniform changes of velocity
  - Inertial frames are non-accelerating. What expt. can you do to tell if you're in an accelerating frame?
- Let's figure out the transformations
  - Consider 2 inertial frames S and S' with S' moving at velocity $\vec{v}$ wrt S.
    - Meaning the spatial origin $\vec{O}'$ moves with velocity $\vec{v}$ compared to $\vec{O}$
    - Choose time origins and space origins (translations!) so that $(t, \vec{r}) = (0, \vec{O})$ coincides with $(t', \vec{r}') = (0, \vec{O}')$
    - Also line up axes so we don't have to worry about rotations





- All clocks in all frames are synchronized
  $$t = t'$$
  To do this, you need infinite speed signalling
- Vector addition gives $\vec{r}' = \vec{r} - \vec{v}t$
  - You can get the inverse by interchanging primed + unprimed and $\vec{v} \Rightarrow -\vec{v}$. (Why?)
- Book uses "standard configuration" to mean velocity along x-axis
  $$t' = t, \quad x' = x - vt, \quad y' = y, \quad z' = z$$
- This is kinematics (description of motion)
  What other kinematic quantities are there, + how do they transform?
  - Velocity: 2 velocity transformations

- Composition of velocities $S \underset{v_1}{\Rightarrow} S' \underset{v_2}{\Rightarrow} S''$ (Passive)

$$x'' = x' - v_2 t' = (x - v_1 t) - v_2 t = x - (v_1 + v_2)t \quad \text{(Standard)}$$

This is the "group multiplication rule" Can you do it in full 3D?

- Velocity of a particle in different frames (active)

Denote by $\vec{u}$, $\vec{v}$ stands for relative velocity of 2 frames

Derivation: $d\vec{r}'/dt' = \vec{u}'$, $d\vec{r}/dt = \vec{u} \Rightarrow \vec{u}' = \vec{u} - \vec{v}$

∴ Only infinite velocities are invariant

• Ex Angle of particle motion $\tan\theta' = \frac{u_y'}{u_x'} = \frac{u_y}{u_x - v} =$

- Accelerations: Work out the steps: $\vec{a}' = \vec{a}$.

• Dynamics are laws governing physical quantities. What are the transformation laws for dynamical quantities

- Mass is an invariant b/c it is an inherent property of an object. Usually drop primes when changing frames
- Momentum: $\vec{p} = m\vec{u}$. Therefore, $\vec{p}' = \vec{p} - m\vec{v}$!
- Energy: Potential energy will transform as required by formula $V(\vec{r}_1, \vec{r}_2, \ldots)$

Kinetic is $k = \vec{p}^2/2m$, so $k' = \vec{p}'^2/2m = k - \vec{v}\cdot p + mv^2/2$ (work out)

- Systems of Particles Total Mass $M = \sum_i m_i$, Center of mass Momentum $\vec{P} = \sum \vec{p}$

Total kinetic energy $K_{tot} = \sum p^2/2m$. Show $K'_{tot} = K_{tot} - \vec{v}\cdot\vec{P} + Mv^2/2$.

But classical mechanics tells us $K_{tot} = K + K_{int}$ with $K = P^2/2M$

But $K' = K - \vec{v}\cdot\vec{P} + Mv^2/2$. Show K_{int} is invariant

• Covariance of Physical Law (+ using invariants)

- Remember that all true laws must be the same form in all frames.

That is, they are co-variant ⟸ lhs + rhs transform the same way.

(Book calls this form-invariant)

- Let's check some conservation laws Δ = final − initial

· Mass. Conservation says $\Delta M = 0$ (even if particles recombine)

This is ok b/c mass is an invariant

- Momentum Conservation. $\Delta \vec{P} = \vec{0}$. In S', this is $\Delta \vec{P}' + \Delta(M\vec{v}) = \vec{0}$
  But M is ~~relativistic~~ Galilean invt + conserved and $\vec{v}$ is a constant relating S and S'
  So $\Delta \vec{P}' = 0$ also.

- Energy conservation. Start by looking at kinetic energy.
  + By conservation of mass + momentum $\Delta K'_{tot} = \cdots = \Delta K_{tot}$.
  For $\Delta E = 0$ and $\Delta E' = 0$, we need $\Delta V = \Delta V'$ for potential energy.
  + In fact, we can argue $V' = V$, ie, V is ~~relativistic~~ Galilean invt.
  The reason is that $K_{int} + V = U$ is total E in CM frame $(\vec{P} = 0)$
  If you always work in center of momentum frame, this is conserved. Intrinsic to system.
  But a quantity defined in a specific frame, ie "CM energy", is invt. K_{int} also invt.
  QED

- Note: if we demand energy conservation be covariant, that requires $\vec{P}$ and M conservation

— Choosing frames + making use of invariants is helpful!

Examples

- A bird sits on the ground in a head wind of speed v. How high can it rise without expending energy? Go to the wind's rest frame. Bird (+ ground) have speed v. Then the bird can clearly rise to height $gy' = v^2/2$. But y is inv.
  In the original frame, the bird now moves horizontally with wind.

- A ball of initial velocity u_i bounces elastically off a very heavy barrier moving with velocity w (in 1D). $\overrightarrow{u_i}$ $\|\overrightarrow{w}$ What's the ball's final velocity?
  Well, in the barrier's frame, $u_i' = -u_f'$ $\overrightarrow{u}\|$. This is an invt
  bc it refers to a specific frame. And in fact $u' = u - w. \Rightarrow u_f - w = -(u_i - w)$

- Consider two lumps of clay, masses m_1 and m_2, velocities u_1, u_2 (in 1D)
  They collide + stick. How much heat is released?
  Note that K_{int} is Galilean invt. In the CM frame, we know $K_{int} = \frac{1}{2}\mu(u_1 - u_2)^2$
  where $\mu = m_1 m_2 / (m_1 + m_2)$ is reduced mass. This must be converted to heat when the balls stick.

— Finally, are Newton's laws covariant? Think about $\vec{F} = m\vec{a}$.