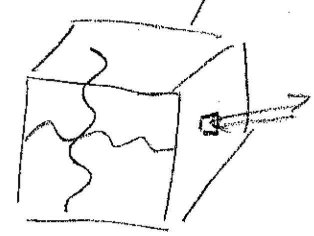
Historical Introduction to Quantum Mechanics

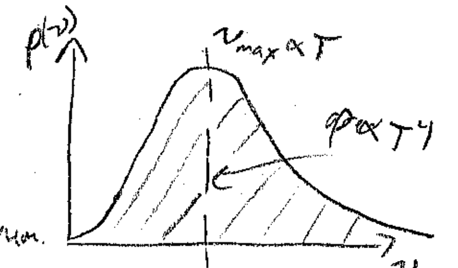
Where does classical physics start to fail?

- **Blackbody (Thermal) Radiation**

  - What are we looking at?
    - This is perfectly thermal radiation from an object or system that absorbs + emits light perfectly (no reflection)
      - + To maintain equilibrium w/ surroundings, absorption + emission must be the same amount. (+ in each frequency range, too)
      - + Real life "blackbodies" (approximate): filament in light bulb, the sun (or star). The most perfect one ever seen is (light from) the early universe
    - + Model: Cubic box with reflecting walls. Light bounces inside. Absorbed or emitted from a small hole



- We want to understand the spectrum of the light. That's energy per frequency interval.



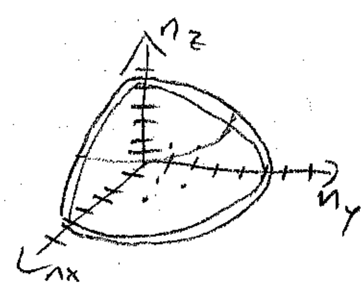
  - + We will look at $\rho(\nu)$, the energy density spectrum.
    $\rho(\nu)d\nu$ = energy density of light in freq. range ν to $\nu + d\nu$
  - + Depends on temperature. Total energy density is

$$\rho = \int_0^\infty d\nu\, \rho(\nu) = \left(\frac{4\sigma}{c}\right) T^4 \qquad \sigma\text{-SI}$$

    This is a version of the Stefan-Boltzmann law. $\sigma = 5.7 \times 10^{-8} \frac{J}{m^2 s K^4}$ = Stefan-Boltzmann constant

  - + The frequency of the peak is $\nu_{max} = \left(\frac{c}{w}\right) T$.
    This is Wien Displacement Law. Usually written $\lambda_{max} T = w$
    $w = 2.9 \times 10^{-3}$ mK.

- Two ingredients: "Density of States" and Energy/mode
  - Density of states = # of different wave modes (wave #s) with same frequency per frequency interval per volume
    + In the box, electromagnetic fields have Dirichlet b.c. (full treatment, see EM theory), specifically potential $\Phi = 0$ at box wall.
      So $\Phi = A \sin(n_x \pi x/L) \sin(n_y \pi y/L) \sin(n_z \pi z/L)$, $n_{x,y,z} \in$ integers
    + Wave vector is $\vec{k} = (\pi/L)(n_x \hat{x} \pm n_y \hat{y} \pm n_z \hat{z})$. Take all as positive
      Therefore $2\pi\nu = (\pi/L) c\sqrt{n_x^2 + n_y^2 + n_z^2} \equiv \frac{c\pi}{L} n$
    + The number of states in range ν to $\nu + d\nu$ is the number of integer lattice points in the spherical shell of appropriate radius. This is approximately 1/8 volume of spherical shell.



$$\frac{1}{8}(4\pi) n^2 dn = \frac{4\pi L^3}{c^3} \nu^2 d\nu$$

    + Also, light waves have 2 polarizations, so total # of modes per volume is

$$n(\nu) d\nu = (8\pi/c^3) \nu^2 d\nu$$

  - The average energy per mode:
    + At a fixed temperature, the probability of a given energy for a single mode follows the Boltzmann factor $P \propto e^{-E/kT}$ k = Boltzmann constant
    + Classically, each mode can have any energy given by the wave amplitude. So

$$\langle E \rangle = \int_0^\infty dE\, E e^{-E/kT} \Big/ \underbrace{\int_0^\infty dE\, e^{-E/kT}}_{\text{total probability}} = kT$$

    + Classically, then, the energy density spectrum is

$$\rho(\nu) d\nu = (kT) n(\nu) d\nu = \frac{8\pi kT}{c^3} \nu^2 d\nu \quad \text{Rayleigh-Jeans}$$

This has the ultraviolet catastrophe: more and more energy density at larger frequencies

– Planck's Law

- Planck's guess about quantization: Each mode is allowed only certain energies $h\nu, 2h\nu, 3h\nu, \ldots = nh\nu$ for positive integer n.
  - + h is a constant of nature called Planck's constant w/ units of Energy × time
  - + The average energy in a mode of frequency ν:

$$\langle E\rangle = \sum_{n=0}^{\infty} nh\nu\, e^{-nh\nu/kT} \Big/ \sum_{n=0}^{\infty} e^{-nh\nu/kT}$$

Well, $\sum_n e^{-nx} = \frac{1}{1-e^{-x}}$, $\sum_n ne^{-nx} = -\frac{d}{dx}\sum_n e^{-nx} = \frac{e^{-x}}{(1-e^{-x})^2}$

so

$$\langle E\rangle = h\nu \Big/ \left(e^{h\nu/kT}-1\right)$$ ⇐ Drops when lowest energy $h\nu$ > classical average.

  - + Then the spectrum is Planck's Law

$$\rho(\nu)\,d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT}-1}\,d\nu$$ goes to 0 at $\nu \to \infty$

- Does this fit what we wanted?
  - + Wien's law comes from $d\rho/d\nu = 0$ to find max

$$3\left(e^{h\nu/kT}-1\right) - \nu\left(\tfrac{h}{kT}\, e^{h\nu/kT}\right) = 0$$

Function only of $h\nu/kT \Rightarrow \nu_{max} \simeq (2.8h/k)T$.

  - + Stefan-Boltzmann comes from integral

$$\rho = \frac{8\pi h}{c^3}\int_0^\infty \frac{\nu^3}{e^{h\nu/kT}-1}\,d\nu = \frac{8\pi k^4T^4}{c^3h^3}\int_0^\infty dx\, \frac{x^3}{e^x-1}\,dx = \left(\frac{8\pi^5}{15}\frac{k^4}{h^3c^3}\right)T^4$$

  - + Matching to Wien + Stefan-Boltzmann constants give

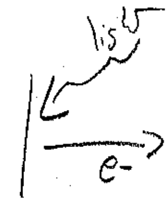
$$h = 6.626\times10^{-34}\ \mathrm{J\,s} \qquad k = 1.381\times10^{-23}\ \mathrm{J/K}$$

- Today, we interpret this as meaning light comes in discrete clumps called photons

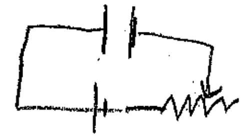
Photoelectric Effect

– Experiment: Light on metal

- light shining on metal can eject electrons.

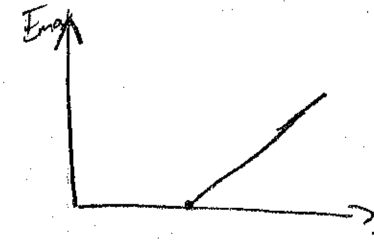


  + You can measure the kinetic energy of the electrons if the plate is in a capacitor



  + Current flow stops when $eV = \max KE$
    This stops ejected e^- before they hit the other plate

- Results: Dependent on frequency of light



  + At low frequency, there is no current.
  + At a critical ν, current turns on, and $E_{max} = h\nu - E_0$

— Interpretation:

- Classically, this is baffling
  + Energy in a wave depends on amplitude, <u>not</u> frequency
    So E_{max} should not depend on frequency
  + You should be able to eject electrons at arbitrarily low freq. if you have a strong enough light source
- Einstein's Quantum interpretation: extend Planck's idea
  + Light comes in individual particles, and electrons interact pretty much just with one at a time ← <u>photons</u>
  + The energy of a photon is $E = h\nu$. When e^- absorbs a photon, it's KE is $h\nu -$ binding energy ← E_0
  + By the way, that's why cell phones don't cause brain cancer
  + The Compton effect leads to the same ideas.
  + But light is also a wave: <u>wave</u> - <u>particle</u> <u>duality</u>

Bohr Atomic Model

- Problems with the atom
  - A hot atomic gas does not emit a continuous spectrum (ie, it does not emit thermal light).
    + Radiation comes out as discrete spectral lines
    + for H, $1/\lambda = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$, $R = 1.097 \times 10^7\ m^{-1}$ = Rydberg constant
  - An e^- orbiting a nucleus accelerates. Accelerating charges radiate. The electron should rapidly fall into the nucleus.
- The Bohr model:
  - Assume angular momentum is quantized in units of $\hbar = h/2\pi$.
    + This says $pr = n\hbar$, $n = 1, 2, 3, \ldots$, or $2\pi r = n(h/p)$
      Suggests an integer # of wavelengths $\lambda = h/p$
    + $\lambda = h/p$ is the <u>de Broglie wavelength</u>. Means that matter has wave + particle nature
  - Use classical mechanics
    + Force balance
$$\frac{e^2}{4\pi\epsilon_0}\frac{1}{r^2} = \frac{mv^2}{r} = \frac{(pr)^2}{mr^3} = \frac{n^2\hbar^2}{mr^3}$$
$$\Rightarrow \quad r = \frac{4\pi\epsilon_0\hbar^2}{me^2}n^2, \quad v = \frac{e^2}{4\pi\epsilon_0\hbar}\frac{1}{n}, \quad n = 1, 2, 3, \ldots$$
    + Total energy
$$E_n = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0}\frac{1}{r} = -\frac{m}{2\hbar}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\frac{1}{n^2} = (-13.6\,eV)\frac{1}{n^2}$$
    + The spectral line wavelengths come from
$$E(\text{photon}) = hc/\lambda = E_n - E_m$$
  - Bohr model is ultimately incorrect but introduces key ideas: quantization of angular momentum + energy and particle-wave duality of matter.