- Collisions, Decays, etc: Putting things Into practice
  - The idea will be to
    1) Use relativistic invariants / dot products to simplify calculations
    2) Use information from the most convenient (often CM) frame
  - • Often, a problem / calculation will call for the initial rest frame of decaying particle or CM frame of 2+ particle collision
  - Our first two examples will show an "obvious" way (according to book) and the relativistic invt way
  - Example 1: 2 Body Decay at Rest

    A particle of mass M decays into particles of mass m_1 and m_2. ($\Lambda \to p + \pi^-$)
    Find energy E_1^* and momentum p_1^* of particle w/mass m_1 in initial rest frame

    • "Separate" Energy and Momentum conservation
      + First, note that $\vec{p}_1^* = -\vec{p}_2^*$. Call the common magnitude p^*
      + You could write that the total energy $Mc^2 = E_1^* + E_2^*$

$$Mc^2 = \sqrt{(p^*c)^2 + m_1^2c^4} + \sqrt{(p^*c)^2 + m_2^2c^4}$$

      Then do a bunch of algebra to find p^*, then E_1
      + A little easier is $E_2^* = \sqrt{(p^*c)^2 + (m_2c^2)^2} = \sqrt{(E_1^*)^2 - (m_1c^2)^2 + (m_2c^2)^2}$

      And

$$Mc^2 = E_1^* + E_2^* \Rightarrow (Mc^2 - E_1^*)^2 = E_1^{*2} - (m_1c^2)^2 + (m_2c^2)^2$$

      Solve

$$E_1^* = (M^2 + m_1^2 - m_2^2)c^2/2M \text{ then find } p^*.$$

    • 4-vector momentum conservation
      + We know that the initial 4-momentum P^μ is conserved

$$P^\mu = p_1^\mu + p_2^\mu.$$

      + If we subtract and square, $p_2^2 = (P - p_1)^2$ or

$$m_2^2c^2 = M^2c^2 + m_1^2c^2 + 2P\cdot p_1$$

      + We can evaluate the scalar product in CM frame

$$P\cdot p_1 = -(Mc)(E_1^*/c).$$ Gives right result.

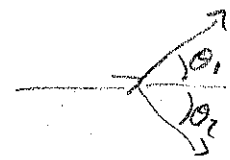
- Example 2: Moving 2 body decay

A π^0 particle of mass m moving at speed u relative to the lab decays into 2 photons that move off at angles θ_1, θ_2 to initial flight path. How does θ_1 depend on E_1?

- Separate Energy + Momentum Conservation
  - For a photon, the magnitude of momentum is E/c
  - Energy conservation says $\gamma m c^2 = E_1 + E_2$
  - Momentum conservation says:

    Along velocity $\gamma m u = (E_1/c)\cos\theta_1 + (E_2/c)\cos\theta_2$

    Perpendicular $(E_1/c)\sin\theta_1 = (E_2/c)\sin\theta_2$



  - First eliminate θ_2, then E_2.

$$(\gamma m c^2 - E_1)^2 = (\gamma m u c)^2 + E_1^2 - 2\gamma m u c E_1 \cos\theta_1$$

After massaging

$$E_1 = \frac{mc^2}{2\gamma\left(1 - \frac{u}{c}\cos\theta_1\right)}$$

- 4-momentum Conservation
  - Again, use $p_2^\mu = P^\mu - p_1^\mu$
  - Squaring, $0 = -m^2c^2 - 2P\cdot p_1$ or $m^2c^2 = 2\gamma m(E_1 - uE_1/c)$

  Easy and direct

- Example 3: Production Thresholds

2 particles collide. What's the minimum energy needed to create some other set of particles? A very important question in expt design.

- A preliminary:
  - Total final state 4-momentum is P_f^μ. Define $P_f^2 = -M^2c^2$.
  - But in CM frame $\vec{P}_f^* = 0$, $P_f^{0*} = \sum_n E_n/c > \sum_n m_n c$

    This means $(P_f^*)^2 \le -(\sum m_n c)^2 \Rightarrow M \ge \sum_n m_n \Leftarrow$ valid in any frame

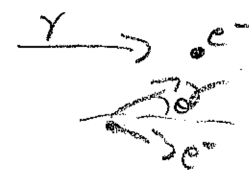
- CM frame: What's the minimum CM frame energy needed?
  - $\vec{p}_1^* + \vec{p}_2^* = 0$. Therefore, $(p_1 + p_2)^2 = -(E_1^* + E_2^*)^2/c^2$
  - 4 momentum conservation: $(E_1^* + E_2^*)^2 = M^2c^4 \ge (\sum m_n)^2 c^4$

- Lab frame: Suppose m_1 is moving but m_2 is stationary.
  - Then $-M^2c^2 = (p_1+p_2)^2 = -m_1^2c^2 - m_2^2c^2 + 2p_1\cdot p_2$

    Since $p_2^\mu = (m_2c, \vec{0})$, $p_1\cdot p_2 = -(m_2c)(E_1/c) = -m_2E_1$
  - Therefore,
    $$E_1 = \frac{1}{2m_2}(M^2 - m_1^2 - m_2^2)c^2 \geq \frac{1}{2m_2}\left[(\Sigma m_n)^2 - m_1^2 - m_2^2\right]c^2$$
  - This shows why Tevatron, LHC collide 2 moving beams (cm frame). At large M^2, $E_1^* \propto \Sigma m_n$ but stationary target $E_1 \propto (\Sigma m_n)^2$.

- Example 4: The Compton Effect

Light strikes a stationary electron. Classically, wavelength is unchanged. Relativistically, we must include electron recoil.



- Electron momenta are p_μ, p'_μ. Photon momenta are q_μ, q'_μ.
  - Conservation
    $$p'^\mu = p^\mu + q^\mu - q'^\mu \Rightarrow -m^2c^2 = (p+q-q')^2$$
    $$= -m^2c^2 + 2p\cdot(q-q') - 2q\cdot q'$$
  - In this frame, $p^\mu = (mc, \vec{0})$, $q^\mu = (E/c, 0, 0, E/c)$,
    $$q'^\mu = (E'/c, 0, E'/c\,\sin\theta, E'/c\,\cos\theta)$$
  - Finally,
    $$m(E-E') = (1/c^2)EE'(1-\cos\theta)$$
- In Quantum mechanics, photon energy is $E = hc/\lambda$.

  h = Planck's constant

  Therefore, the light changes wavelength
  $$\lambda' - \lambda = (h/mc)(1-\cos\theta)$$
  - h/mc is called the electron's Compton wavelength