

PHYS-3301 Homework 9 Due 16 Nov 2011

This homework is due in class on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. Energy Release Efficiencies

In this question, you will find the fraction of mass energy released in a couple of nuclear reactions. It will be useful for you to know that one atomic mass unit (aka one *dalton*) is $930 \text{ MeV}/c^2$ and also $1.7 \times 10^{-27} \text{ kg}$ (both to two significant figures).

- (a) In a nuclear reactor, a slow (nonrelativistic) neutron of mass $940 \text{ MeV}/c^2$ hits a Uranium-235 nucleus (mass 240 dalton), which breaks into various fragments. This *fission* (splitting) reaction releases 200 MeV of energy into heat, which the reactor can then convert to work. What fraction of the initial mass energy is converted to useful heat?
- (b) The sun produces heat through the *fusion* of light nuclei into heavier ones. Most of this “hydrogen-burning” converts 6 protons (mass $940 \text{ MeV}/c^2$) into a He-4 nucleus (mass 4.0 dalton), 2 protons, 2 positrons (aka anti-electrons) of mass $0.51 \text{ MeV}/c^2$, two neutrinos (essentially zero mass), and photons (zero mass). The two positrons immediately find two electrons and annihilate into photons. Assume that all of the energy above the rest energy of the final state particles is converted to heat. What fraction of the initial mass energy is converted to heat?

2. Electron Absorbing a Photon

Imagine an electron (mass m) at rest. It is hit by a photon of energy E and absorbs it completely. The final state of the system is just an electron, now moving off in some direction. You will prove in two different ways that this is *impossible* according to conservation of 4-momentum.

- (a) *First method:* Consider the reference frame where the final electron is at rest. In this frame, what is the energy of the final electron? Is the energy of the initial electron greater or less than this amount (assuming the photon has nonzero 4-momentum)? You should see that energy cannot be conserved in this frame, which means 4-momentum is not conserved in this or any other frame.
- (b) *Second method:* Conservation of 4-momentum says $P_i^\mu = P_f^\mu$, where P_i is the total initial momentum and P_f is the total final momentum. Show by taking the Lorentz-invariant square of this equation that 4-momentum can be conserved only if the initial photon energy is $E = 0$.