PHYS-3301 Homework 6 Due 26 Oct 2011

This homework is due in class on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **A Moving Object** *Barton 6.6 plus*

   A relativistic car moves along the x axis, passing $x = 0$ at $t = 0$. Its velocity is $u(t) = dx/dt = c/\cosh(\omega t)$. **Important:** In the following, you will find some integrals involving hyperbolic trig functions. You may use computer programs such as Maple or Mathematica to do them ***only*** if you *cite the program you use* and *check the result of the indefinite integral by differentiating it and seeing that you get the integrand.*

   (a) Sketch the worldline of the car on a spacetime diagram. Can a real car follow this path exactly?

   (b) Find the relation between the time t and the car's proper time τ. Choose integration constants so that $\tau = 0$ when $t = 0$. Be careful taking signs if you take a square root.

   (c) Find u as a function of τ and then the car's position x as a function of τ. *Hint:* Remember that $u = dx/dt$, not $dx/d\tau$. Once you have $u(\tau)$, you can get x by integrating $dx/d\tau = u(dt/d\tau)$. (Think about the chain rule.)

   (d) Since the car moves, it doesn't experience as much proper time as coordinate time t. Find the total time lag Δ over all time, which is defined as

   $$\Delta = \lim_{t\to\infty} [t - \tau(t)] - \lim_{t\to-\infty} [t - \tau(t)] \ .$$

2. **Velocity Addition**

   In the lecture notes, we argued that a boost by v_1 along x followed by a boost of v_2 also along x is the same as a boost along x by

   $$v_3 = \frac{v_1 + v_2}{1 + v_1 v_2/c^2} \ , \tag{1}$$

   but we didn't finish the proof.

   (a) We needed to show that $\gamma(v_3) = \gamma(v_1)\gamma(v_2)(1 + v_1 v_2/c^2)$. To do this, prove the following:

   $$1 - \frac{v_3^2}{c^2} = \frac{(1 - v_1^2/c^2)(1 - v_2^2/c^2)}{(1 + v_1 v_2/c^2)^2} \ , \tag{2}$$

   using v_3 as given in (1). This is $1/\gamma(v_3)^2$, so it proves what we want.

   (b) If $v_1 \leq c$ and $v_2 \leq c$, can v_3 ever exceed c?

   (c) Give a quick argument relating this velocity addition to the transformation of particle velocities to prove (1) indirectly.

3. **Relative Velocities**

   I'm sitting in a lab in a particle accelerator, and I see two protons approaching each other on the x axis. One has velocity $u_1 = 3c/4$ and the other has velocity $u_2 = -3c/4$ in my frame.

   (a) At what speed do I see them approach each other? To think about this, you might want to find their positions $x_1(t)$ and $x_2(t)$ and then figure out $d(x_2 - x_1)/dt$ just to get things straight in your head.

(b) You might be surprised at your answer, since it should be bigger than c. Is anything real moving faster than c?

(c) Find the velocity u' of one particle relative to the other (that is, the velocity of one particle in the rest frame of the other).