PHYS-3301 Homework 5 Due 19 Oct 2011

This homework is due in class on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Twins Again** *Inspired by Barton 5.6*

   Remember our twins Frankie and Fannie. Frankie stays on earth, while Fannie flies off (at birth) to a star 20 lightyears away at constant speed $u = 4c/5$. When she reaches the star, Fannie turns around immediately and returns to the earth at the same speed. This problem will start to help you see why Frankie and Fannie are different. *Hint:* It will probably help you to look at the solution set for assignment 4.

   (a) The earth and the star are at positions $x = 0$ lightyears and $x = 20$ lightyears in the earth's inertial reference frame. Write Fannie's path as a function $x(t)$, where t is measured in the earth's reference frame.

   (b) Every ten years on his birthday, Frankie sends a light signal to Fannie. Does Fannie receive any of the signals before she reaches the star and turns around?

   (c) Draw a spacetime diagram in the earth's reference frame. Show Fannie's worldline and the worldlines of the light signals.

   Note: Light speed can be written in appropriate units as $c = 1$ lightyear/year. Those units might make life easier.

2. **Space Race Again**

   Recall the race between the $\times$-wing and "The Fastest Hunk of Junk in the Galaxy" (TFHoJitG) from the previous assignment. Remember, the $\times$-wing flies at speed $c/4$ and TFHoJitG can fly at $c/2$. Three events happen in the race:

   A. The $\times$-wing takes off at the starting point.
   B. A while later, TFHoJitG takes off at the starting point.
   C. They arrive at the ending point 10 light-minutes away at the same time.

   This race happens in only one dimension of space. *Note:* A light-minute is the distance light can travel in one minute (in a vacuum).

   (a) Draw a spacetime diagram that shows the worldlines of both ships from the reference frame of the fixed starting and ending points. Also show the worldlines of the starting and ending points. Label the worldlines clearly (you may use colors and a legend) and draw the axes perpendicular to each other.

   (b) Now draw a spacetime diagram in the reference frame of the $\times$-wing. Again show the worldlines of the two ships, starting point, and end point, and label them clearly. Again, draw the space and time axes perpendicularly to each other.

   (c) Finally, draw a spacetime diagram in the rest frame of the starting and ending poings (space and time axes perpendicular). On this diagram, draw the space and time axes associated with the rest frame of the $\times$-wing.

3. **Lightcone Coordinates**

   We are used to labeling time t as distinct from space x, y, z. But the Lorentz transformations tell us that there is less difference than we think, so we might try some other set of labels. In this problem, define the lightcone coordinates

$$x_+ = x + ct \ , \quad x_- = x - ct \ . \tag{1}$$

We can use these as the independent variables to describe physics if we want, writing x and t as functions of x_+ and x_-.

(a) On a spacetime diagram with the x and t axes perpendicular, draw axes for x_+ and x_-. Why are x_+ and x_- called lightcone coordinates (you should see why on the diagram)?

(b) Find the formula for the invariant interval δs^2 in terms of δx_+ and δx_-.

(c) Consider a reference frame S' moving at speed v in standard configuration with our original reference frame S. Define new lightcone coordinates

$$x'_+ = x' + ct' \ , \quad x'_- = x' - ct' \ . \tag{2}$$

Use the Lorentz transformations to find x'_+ and x'_- in terms of x_+ and x_-.