PHYS-3301 Homework 2 Due 28 Sept 2011

This homework is due in class on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

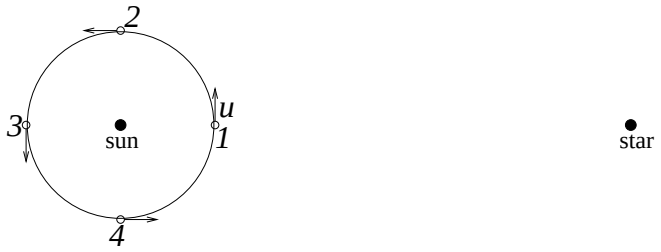
Helpful Information: Remember that the speed of light in vacuum is $c = 3 \times 10^8$ m/s.

1. **Stellar Aberration**

   In this problem we will explore more the aberration of starlight that was measured as far back as 1725. In this problem, all speeds of objects are small compared to the speed of light, so you are free to use Newtonian/Galilean relativity.

   (a) First, to get a feel for how this works, consider the following situation. You're driving in a car, and it's raining. Relative to the fixed earth, the rain falls straight down with speed w, and you drive at speed u. At what angle from the vertical do you see the rain falling?

   (b) Now, suppose there is a star straight overhead compared to your telescope. However, the earth is at position 1 in its orbit around the sun (see the figure below), where the orbital speed of the earth is approximately $u = 3000$ m/s. At what angle from the vertical must I, standing on the earth, aim my telescope so that light from the star falls down the telescope tube? You may ignore the rotational speed of the earth's surface, which is much smaller than the earth's orbital speed. *Hint:* Recall that $\tan\theta \approx \sin\theta \approx \theta$ for small angles θ.



   Note that the figure is not to scale; the star is far enough away that it is effectively directly overhead (at the appropriate time of day) no matter where the earth is in its orbit.

   (c) At which point(s) as labeled in the figure above is this angle of aberration maximized? At which points is it minimized?

2. **Global Positioning System**

   This problem will investigate how important relativistic effects are to one piece of technology that you may use in your daily life (like the Winnipeg Transit buses, which use GPS to identify the next stop). A GPS unit works by receiving time signals from multiple satellites and using that to calculate its position based on the distances to those satellites (which is given by c multiplied by the time the signal takes to reach the GPS from the satellite).

   (a) GPS satellites orbit at a height of approximately $r = 30,000$ km from the center of the earth. Find the ratio u/c of the orbital speed of the satellite to the speed of light, first in terms of Newton's constant G, the mass of the earth $M_\oplus$, and orbital radius r, and then as a pure number. *Reminder:* To one significant digit, $G = 7 \times 10^{-11}$ m^3/kg/s^2 and the mass of the earth is $M_\oplus = 6 \times 10^{24}$ kg.

(b) GPS claims to be able to locate a receiver to within a radius of about 1 m. Considering that the distances used to locate a receiver are calculated by how long a light signal takes to travel between the satellite and the receiver, what error δt in the time signal sent by the satellite would translate into an error of 1 meter in position? You may work approximately, ignoring numerical factors of order one.

(c) Because no signal can travel faster than c in special relativity, it is not possible to synchronize all clocks in all reference frames. Specifically, if a time t elapses on a clock in frame S, then the time t' elapsed in frame S' differs by

$$t' - t \approx \frac{v^2}{c^2} t \ ,$$

where v is the relative speed of the two frames and $v \ll c$. If the GPS did not account for special relativity, this would be an error δt in the time signal as in part (b). How long would it take (elapsed time t) for δt to become large enough to give an error in location of more than a meter? *Note:* Once again, we are dropping numerical constants of order 1 and working with one significant digit.

3. **Homework Comments**

The following questions are **ungraded**, but your answers are greatly appreciated.

(a) On a scale of 1 to 10, with 1 being very easy, 10 very difficult, and 5 the average of homeworks from your physics classes last year, how difficult was this assignment?

(b) On a scale of 1 to 10, with 1 being very short, 10 very long, and 5 the average of homeworks from your physics classes last year, how long was this assignment?