PHYS-3301 Homework 10 Due 23 Nov 2011

This homework is due in class on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **SN1987A and Neutrino Masses**

   On 23 Feb 1987, astronomers were startled by the observation of a new supernova in the Large Magellanic Cloud, a satellite galaxy of our Milky Way. However, the first observation of this supernova was several hours earlier by the detection of neutrinos, which was confirmed by two detectors. (The neutrinos arrived before the light because light is trapped for a while by all the matter inside the exploding star.) The fact that the neutrinos all arrived within a few seconds of each other after traveling for more than 100,000 lightyears allows us to put tight constraints on the mass of the neutrino. This problem will guide you through a real calculation of this limit.

   Note: In this problem, you will forget that you ever heard about neutrinos moving faster than the speed of light. Assume that (as is most likely anyway) neutrinos are normal particles with a small mass m.

   (a) Show that a neutrino with energy $E \gg mc^2$ has a speed approximately given by

   $$\frac{|\vec{u}|}{c} \approx 1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2 . \tag{1}$$

   Hint: We gave formulas in class for energy both in terms of the spatial momentum and in terms of the speed. Try looking at those. Then you will need to make an expansion in powers of mc^2/E.

   (b) Light (once free of the matter in the supernova) takes a time $t_0 = 5.3 \times 10^{12}$ s to travel from SN1987A to the earth. How long would a neutrino of energy E take to reach earth from the supernova? Work to the lowest non-trivial order in mc^2/E and give the answer in terms of t_0, m, c, and E.

   (c) The Kamioka detector in Japan detected several neutrinos. The first arrived with energy 21.3 MeV, and another with energy 8.9 MeV arrived 0.303 s later. Assuming that the second neutrino left the supernova no more than 1 s before the first, what is the maximum neutrino mass m? For simplicity, we are ignoring the possible error in the measurements. *Hint:* The observation time of each neutrino is its emission time plus its travel time; take the difference of these and be careful of signs.

2. **Mandelstam Variables** *Based on Barton 11.8*

   Imagine a process where a particle of mass m_1 and 4-momentum p_1^μ collides with a particle of mass m_2 and 4-momentum p_2^μ. After the collision, there are particles of mass m_3 and momentum p_3^μ and mass m_4 and momentum p_4^μ. To describe this scattering process, particle physicists will often define the *Mandelstam variables* s, t, u as

   $$s = -(p_1 + p_2)^2 \ , \quad t = -(p_1 - p_3)^2 \ , \quad u = -(p_1 - p_4)^2 \ , \tag{2}$$

   where the square is the relativistic dot product of each 4-vector with itself.

   (a) Show that $s + t + u = (m_1^2 + m_2^2 + m_3^2 + m_4^2)c^2$. *Hint:* 4-momentum conservation allows you to write $p_3^\mu + p_4^\mu - p_2^\mu = p_1^\mu$.

(b) Show that $s = (E^*/c)^2$, where E^* is the total energy in the CM frame.

3. **Photo-Production of Electrons and Positrons** *Based on a problem by J. D. Jackson*

   The universe is filled with photons left over from the Big Bang which have a typical energy 2.5×10^{-4} eV. This is called the cosmic microwave background (CMB) radiation. Suppose another photon of energy E hits a typical CMB photon head-on. What is the minimum value of E required for the two photons to produce an electron-positron pair? Electrons and positrons have mass $m = 0.5$ MeV$/c^2$.