PHYS-3301 Homework 1 Due 21 Sept 2011

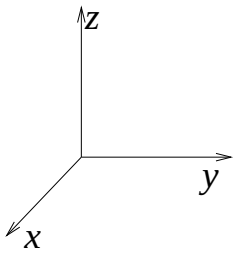
This homework is due in class on the due date. If you wish to turn it in ahead of time, you may email a PDF or give a hardcopy to Dr. Frey.

1. **Reflections and Parity**

   This question is about the group $O(3)$, which we saw in class is related to 3D rotations.

   (a) Explain why the matrix $P_{xy} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ represents reflections through the xy plane. Then show that it is orthogonal, so it is a part of the group $O(3)$.

   (b) If we start with a coordinate system with axes as shown in the plot below, draw the coordinate axes after acting with a reflection P_{xy}. Is it possible to obtain the new axes by rotating the initial axes?



   (c) Write down matrices P_{yz} and P_{xz} corresponding to reflections through the yz and xz planes. Find the *parity* matrix $P = P_{xy}P_{yz}P_{xz}$. What kind of reflection is P? Can you find a rotation that turns P_{xy} into P?

   (d) Evaluate the determinant of the parity matrix and also the rotation matrix given in the class notes for rotations around the z axis. Note that we can now define a new group $SO(3)$ of *special* orthogonal matrixes that contain only matrices with determinant equal to 1. Are there any reflections in this group?

   Parity is a symmetry of Newtonian mechanics, electromagnetism, and most physics you'd think of. However, it isn't a symmetry of all of subatomic physics! The discovery that parity is not a symmetry won a Nobel prize.

2. **CM Frame**

   Consider n particles (labeled $i = 1, \ldots n$) in motion and define the total mass $M = \sum_i m_i$ and center of mass velocity $\vec{U} = (1/M)\sum_i m_i \vec{u}_i$.

   (a) Show that the total kinetic energy can be written as $K_{tot} = \vec{P}^2/2M + K_{int}$, where $\vec{P} = M\vec{U}$ is the total momentum and K_{int} is the kinetic energy measured in the rest frame of the center of mass (in this class, we will call this frame the CM frame for "center of mass" or "center of momentum"). Write K_{int} solely in terms of quantities that are invariant under Galilean transformations, which proves that it is invariant.

   (b) In the case that $n = 2$, show that we can further write $K_{int} = \mu\vec{u}^2/2$, where $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass and $\vec{u}$ is the relative velocity of the 2 particles.

3. **Choosing Frames Wisely**

   In both parts, clearly state what inertial reference frame you use to solve the problem.

(a) *Barton 2.2 rephrased* A river flows at 5 km/hr, and a boat in it can move 8 km/hr relative to the water. As the boat moves upstream, the driver hears a splash but only realizes that it was the life preserver falling overboard 15 minutes later. The driver turns around and heads to retrieve the life preserver. How soon can the boat catch up to the life preserver?

(b) *Barton 2.10 rephrased* A cannonball is launched in an arc with velocity $\vec{u}$. At the top of its trajectory, a chemical charge in it explodes into two parts of masses m_1 and m_2 that separate in the horizontal direction only. The explosion releases energy E, which essentially all goes into the kinetic energy of the cannonball pieces. Show that they are separated by a distance $u_y/g\sqrt{2E(m_1+m_2)/m_1m_2}$ when they land, where u_y is the initial vertical component of the velocity.

4. **Homework Comments**

The following questions are **ungraded**, but your answers are greatly appreciated.

(a) On a scale of 1 to 10, with 1 being very easy, 10 very difficult, and 5 the average of homeworks from your physics classes last year, how difficult was this assignment?

(b) On a scale of 1 to 10, with 1 being very short, 10 very long, and 5 the average of homeworks from your physics classes last year, how long was this assignment?